Gravitation

ELEMENTARY

Q.1 (4)

$F \propto \frac{1}{r^2}$. If r becomes double then F reduces to $\frac{F}{4}$

Q.2 (1)

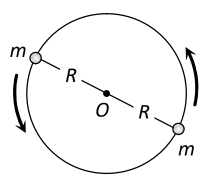
k represents gravitational constant which depends only on the system of units.

Q.3 (3)

Centripetal force provided by the gravitational force of attraction between two particles

i.e. $\frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2}$

$\Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$



Q.4 (4)

Q.5 (4)

Time period of simple pendulum $T = 2\pi\sqrt{\frac{l}{g'}}$

In artificial satellite $g' = 0$

$\therefore$ T = infinite.

Q.6 (2)

Because acceleration due to gravity increases

Q.7 (2)

$$\frac{g'}{g} = \frac{M'}{M}\left(\frac{R}{R'}\right)^2 = \left(\frac{2M}{M}\right)\left(\frac{R}{2R}\right)^2 = \frac{1}{2}$$

$$\Rightarrow g' = \frac{g}{2} = \frac{9.8}{2} = 4.9 \text{ m/s}^2$$

Q.8 (2)

$g = \frac{GM}{R^2}$. If radius shrinks to half of its present value then g will becomes four times.

Q.9 (1)

$$g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

Q.10 (1)

$$I = \frac{-dV}{dx}$$

If V = 0 then gravitational field is necessarily zero

Q.11 (1)

$$U = -\frac{GMm}{r}$$

$$\Rightarrow 7.79 \times 10^{28} = \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22} \times 6 \times 10^{24}}{r}$$

$\Rightarrow$ r = 3.8 × 10^8 m

Q.12 (4)

$$\Delta U = U_2 - U_1 = \frac{mgh}{1+\frac{h}{R_e}} = \frac{mgR_e}{1+\frac{R_e}{R_e}} = \frac{mgR_e}{2}$$

$$\Rightarrow U_2 - (-mgR_e) = \frac{mgR_e}{2} \Rightarrow U_2 = -\frac{1}{2}mgR_e$$

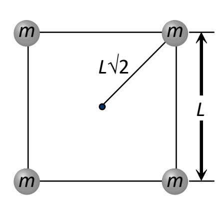
Q.13 (4)

Change in potential energy in displacing a body from r_1 to r_2 is given by

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = GMm\left(\frac{1}{2R} - \frac{1}{3R}\right) = \frac{GMm}{6R}$$

Q.14 (1)

Potential at the centre due to single mass = $\frac{-GM}{L/\sqrt{2}}$



Potential at the centre due to all four masses

$$= -4\frac{GM}{L/\sqrt{2}} - 4\sqrt{2}\frac{GM}{L}$$

$$= -\sqrt{32}\times\frac{GM}{L}$$

Q.15 (3)

Q.16 (2)

$$v_e = \sqrt{2gR} \text{ and } v_0 = \sqrt{gR}$$

$$\therefore\ \sqrt{2}v_0 = v_e$$

Q.17 (2)

$$v = \sqrt{\frac{GM}{r}} \text{ if } r_1 > r_2 \text{ then } v_1 < v_2$$

Orbital speed of satellite does not depends upon the mass of the satellite

Q.18 (1)

$$\mathbf{v_0 = \sqrt{\frac{GM}{(R+h)}}}$$

Q.19 (2)
Time period of communication satellite T_c = 1 day
Time period of another satellite = T_s

$$\frac{T_s}{T_c} = \left(\frac{r_s}{r_c}\right)^{3/2} = (4)^{3/2} \Rightarrow T_s = T_s \times (4)^{3/2} = 8 \text{ days}$$

Q.20 (2)

Q.21 (3)
Areal velocity of the planet remains constant. If the areas A and B are equal then $t_1 = t_2$.

Q.22 (4)

$$T^2 \propto r^3 \Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

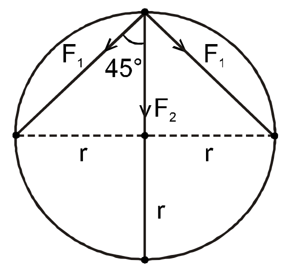
Q.23 (3)

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (1)
Net force is towards centre

$$F_{net} = F_2 + \frac{2F_1}{\sqrt{2}}$$

$$= F_2 + \sqrt{2}F_1$$



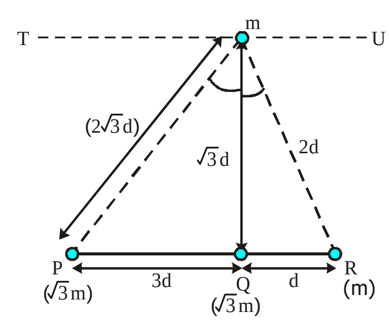
$$\left.\begin{aligned} F_2 &= \frac{Gm^2}{(2r)^2} = \frac{Gm^2}{4r^2} \\ F_1 &= \frac{Gm^2}{(\sqrt{2}r)^2} = \frac{Gm^2}{2r^2} \end{aligned}\right\} \to F_{net} = \frac{Gm^2}{4r^2} + \frac{\sqrt{2}Gm^2}{2r^2}$$

$$= \frac{mv^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{Gm}{4r}(1+2\sqrt{2})} \textbf{ Ans.}$$

Q.2 (3)
In horizontal direction

$$\text{Net force} = \frac{G\sqrt{3}\,mm}{12d^2}\cos 30° - \frac{G\,m^2}{4d^2}\cos 60°$$



$$= \frac{G\,m^2}{8d^2} - \frac{G\,m^2}{8d^2} = 0$$

in vertical direction

$$\text{Net force} = \frac{G\sqrt{3}\,m^2}{12d^2}\cos 60° + \frac{G\sqrt{3}\,m^2}{3d^2} + \frac{G\,m^2}{4d^2}$$

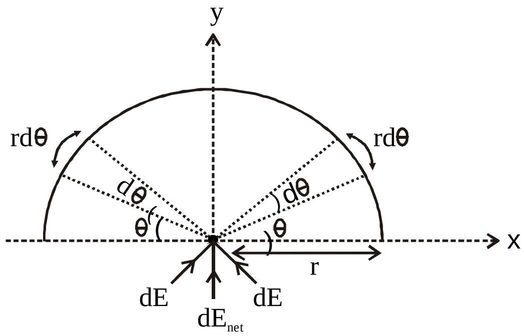
cos30°

$$= \frac{\sqrt{3}G\,m^2}{24d^2} + \frac{\sqrt{3}G\,m^2}{3d^2} + \frac{\sqrt{3}G\,m^2}{8d^2}$$

$$= \frac{\sqrt{3}G\,m^2}{d^2}\left[\frac{1+8+3}{24}\right] = \frac{\sqrt{3}G\,m^2}{2d^2} \text{ along SQ}$$

Q.3 (4)

$dE_{net} = 2dE\sin\theta$



$$= 2\,\frac{Gdm}{r^2}\sin\theta$$

$$= 2G\,.\,\frac{\lambda r d\theta}{r^2}\sin\theta$$

$$= \frac{2G\lambda}{r}\sin\theta\, d\theta$$

$$E_{net} = \int dE_{net}$$

$$= \int_0^{\pi/2}\frac{2G\lambda}{r}\sin\theta\; d\theta$$

$$= \frac{2G\lambda}{r}$$

$$\lambda = \frac{m}{\ell} \text{ and } r = \frac{\ell}{\pi}$$

$$E_{net} = \frac{2Gm\pi}{\ell^2}$$

Along + y axis **Ans.**

Q.4 (2)

$$F = \frac{G\,m_1 m_2}{r^2}$$

$$F' = \frac{G\,m_1 m_2}{(2r)^2} = \frac{F}{4}$$

Q.5 (4)

For point 'A' :

For any point outside, the shells acts as point situated at centre.

$$\text{So, } F_A = \frac{G\,(M_1+M_2)}{p^2}\,m$$

For point 'B' :

There will be no force by shell B.

$$\text{So, } F_B = \frac{GM_1 m}{q^2}$$

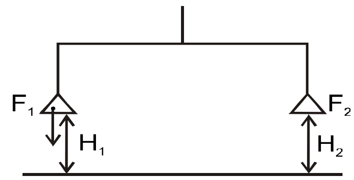
For point 'C' :

There will be no gravitational field.

So, $F_C = 0$

Q.6 (2)

$$\text{Net torque} = F_2\,.\,\frac{\ell}{2} - F_1\,.\,\frac{\ell}{2}$$



$$= (F_2 - F_1)\,\frac{\ell}{2}$$

$$F_2 = mg_{H_2} = mg\left\{1 - \frac{2H_2}{R}\right\}$$

$$F_1 = mg_{H_1} = mg\left\{1 - \frac{2H_1}{R}\right\}$$

$$\tau = (F_2 - F_1)\,\frac{\ell}{2} = \frac{mg\,(H_1 - H_2)\,\ell}{R} \text{ Ans.}$$

Q.7 (3)

$$g = \frac{GM}{R^2} = \frac{G\,\frac{4}{3}\pi R^3\rho}{R^2}$$

$$\rho = \frac{3g}{4\pi R G}$$

Q.8 (4)

$$\frac{g}{4} = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

$$2 = 1 + \frac{h}{R_e}$$

$h = R_e$

Q.9 (2)

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{\left(1+\frac{1}{2}\right)^2} = \frac{4g}{9}$$

decrease = g - g'

$$= g - \frac{4g}{9} = \frac{5g}{9}$$

Q.10 (2)

$$\frac{g}{4} = \frac{GM_e}{(R_e+h)^2}$$

$$\frac{GM_e}{4R^2} = \frac{GM_e}{(R_e+h)^2}$$

$R_e + h = 2R_e$

$R_e = h$

Q.11 (3)

Due to rotation of earth

$g_{eff} = g - \omega_g^2 R \sin^2\theta$

$g \downarrow$

$\therefore$ weight $\downarrow$

Q.12 (3)

$$g_h = g\left(1-\frac{r}{R}\right)$$

$$g_h = \frac{GM}{R^2}\left(1-\frac{r}{R}\right)$$

$$\frac{GMr}{R^3} = \text{constant} \Rightarrow d \propto \frac{1}{r}$$

Q.13 (1)

According to given condition

$$g - \omega^2 R_e = g\left(1-\frac{d}{R_e}\right)$$

$$\frac{\omega^2 R_e^{\,2}}{g} = d$$

Q.14 (2)

$$g = \frac{GM_e}{R_e^{\,2}} = \frac{GM_e'}{(5R_e)^2}$$

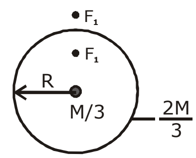
$$\frac{\frac{4}{3}\pi R_e^{\,3}\rho}{R_e^{\,2}} = \frac{\frac{4}{3}\pi(5R_e)^3\rho'}{(5R_e)^2}$$

$\rho = 5\rho'$

Q.15 (2)

$$g' = \frac{G(2M_e)}{(2R_e)^2} = \frac{g}{2}$$

$$T = 2\pi\sqrt{\frac{l}{g}} = 2$$



New time period $T' = 2\pi\sqrt{\frac{l}{g/2}} = 2\sqrt{2}$

Q.16 (2)

$$g = \frac{GM}{R^2}$$

Q.17 (3)

Q.18 (2)

$dv = -Edr$

$$= \frac{k}{r}\,dr$$

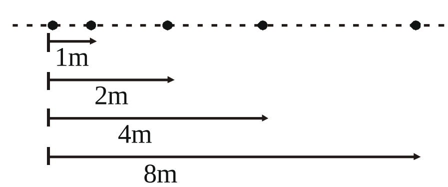
Integrating both sides

$$[v]_{v_i}^{v} = k\;k[\ell n\, r]_{d_1}^{r}$$

$$v - v_i = k\,\ell n\frac{r}{d_i}$$

$$v = v_i + k\,\ell n\frac{r}{d_i}$$ **Ans.**

Q.19 (4)



$v = v_1 + v_2 + v_3 + v_4 + ...$

$$= -\frac{Gm}{1} - \frac{Gm}{2} - \frac{Gm}{2} - \frac{Gm}{8} - \frac{Gm}{16} - \frac{Gm}{32} - ....$$

$$= -Gm\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.......\right) \rightarrow$$ G.P. of infinite series

$= -m\left(\frac{1}{1-1/2}\right)$

$v = -Gm\,(2)$
$= -2Gm$

Q.20 (3)

Initial total energy = Initial kinetic energy + initial potential energy

$= \frac{1}{2}m\,(0)^2 + \left(-\frac{GMm}{R_0}\right) = -\frac{GMm}{R_0}$

Total energy, when it reaches the surface of earth =

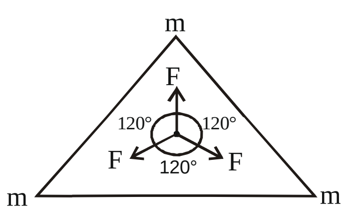
$\frac{1}{2}\,mv^2 + \left(-\frac{GMm}{R}\right)$

Applying energy conservation,

$\frac{1}{2}\,mv^2 - \frac{GMm}{R} = -\frac{GMm}{R_0}$

$v = \sqrt{2GM\left\{\frac{1}{R}-\frac{1}{R_0}\right\}}$ **Ans.**

Q.21 (4)



Due to geometry net force is zero.

Q.22 (4)

Q.23 (1)

$\frac{1}{2}mv'^2 = 2\times\frac{1}{2}mv^2$

$v' = \sqrt{2}\,v_0$

$v' = v_e$

$\therefore$ so escape.

Q.24 (3)

$v_0 = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{\frac{2G\rho\frac{4}{3}\pi R_e^{\,3}}{R_e}} = \sqrt{2G\rho\frac{4}{3}\pi R_e^{\,2}}$

$v' = \sqrt{2G\rho\frac{4}{3}\pi\left(2R_e\right)^2} = 2v_0$

Q.25 (1)

$g_A = \frac{G\frac{4}{3}\pi R_A^3\rho}{R_A^2}$; $g_B = \frac{G\frac{4}{3}\pi R_B^3\rho}{R_B^2}$

$R_A = 2R_B$

$\Rightarrow g_A = 2g_B$

$V_{es} = \sqrt{2gR}$

$(V_{es})_A = \sqrt{2g_AR_A} = 2\sqrt{2g_BR_B}$

$(V_{es})_B = \sqrt{2g_BR_B}$

$\frac{v_A}{v_B} = 2$

Q.26 (2)

$V_P = -\frac{GM}{R}(-ve)$

as $R\downarrow$ $\frac{GM}{R}\uparrow$ but due to –ve it decreases.

Q.27 (3)

$V_e = \sqrt{\frac{2GM}{R}}$

$V = KV_e = K\sqrt{\frac{2GM}{R}}$

Initial total energy $= \frac{1}{2}\,mv^2 - \frac{2GMm}{R}$

$= \frac{1}{2}\,m.K^2\,\frac{2GM}{R} - \frac{2GMm}{R}$

Final total energy $= \frac{1}{2}\,m0^2 - \frac{2GMm}{x}$

Applying energy conservation :

$\frac{1}{2}\,mx^2.\,\frac{2GM}{R} - \frac{2GMm}{R} = 0 - \frac{2GMm}{x}$

$\frac{1}{x} = \frac{1}{R} - \frac{x^2}{R}$

$x = \frac{R}{1-k^2}$ **Ans.**

Q.28 (4)

Q.29 (3)

$$\Delta U = \frac{mgh}{1+h/R}$$

Substituting R = 5h

We get $\Delta U = \frac{mgh}{1+1/5} = \frac{5}{6}mgh$

Q.30 (2)

$$W = \frac{GmM}{R} - \frac{GmM}{nR+R}$$

$$= \frac{GMm}{R}\left[1-\frac{1}{nH}\right] = \frac{GMm}{R}\left[\frac{n}{n+1}\right] = mgR\left(\frac{n}{n+1}\right)$$

Q.31 (4)

$$\frac{GM_P m}{R_P} = 54 \Rightarrow \frac{GM_P}{R_P}\times 3 = 54$$

$$\frac{GM_P}{R_P} = 18$$

$$v_e = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{2\times 18} = 6\,m/sec$$

Q.32 (3)

$$\frac{1}{2}mv^2 = \frac{GM_e m}{Re+h} = \frac{g\,Re^2\,m}{Re+4Re}$$

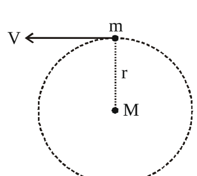
$$\frac{1}{2}mv^2 = \frac{Mg\,Re}{5}$$

Q.33 (2)

$$F = \frac{Gm_1m_2}{r^2}$$

Q.34 (2)

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$



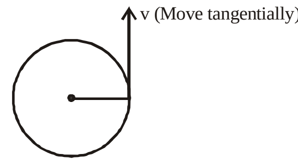
$$v = \sqrt{\frac{GM}{r^2}}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{GM}} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{G\rho\times\frac{4}{3}\pi r^3}}$$

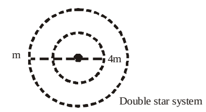
$$T \propto \frac{1}{\sqrt{\rho}}$$ **Ans.**

Q.35 (4)
Net force on the package is zero hence it will revolve around the earth and never reach to earth surface.

Q.36 (2)



Q.37 (1)



$$\frac{k_1}{k_2} = \frac{1/2I_1\omega^2}{1/2I_2\omega^2} = \frac{m_2}{m_1}$$

Q.38 (3)
(1) cavity at center, field is zero
(2) Arc of ellipse
(3) for escape T.E. = 0
(4) Notes.

Q.39 (4)

P.E. $= -\frac{Gm_1m_2}{r}$

T.E. $= -\frac{Gm_1m_2}{2r}$

K.E. $= +\frac{Gm_1m_2}{2r}$

Q.40 (1)
According to kepler's law applying angular momentum conservation $m_1v_1r_1 = m_2v_2r_2$ V_{max} is (a) ans.

Q.41 (1)
$w_e = 50 \times 10 = 500$ N
$w_p = 50 \times 5 = 250$ N
Hence option A is correct

Q.42 (3)

$$\frac{1}{2}mv^2 = mgh = \frac{mGM}{R^2}\times 90 \quad ....(1)$$

$$\frac{1}{2}mv^2 = \frac{mG\left(\frac{1}{10}M\right)}{\left(\frac{R}{3}\right)^2}\times G_1 \quad ....(2)$$

from (1) & (2)

$$m\frac{GM}{R^2}\times 90=\frac{9}{10}\frac{mGM}{R^2}\times h_1 \Rightarrow h_1=100m$$

Q.43 (2)
Gravity of earth vanishes means there is no centripetal force that is gravitational force.

Q.44 (1)

$$E=-\frac{Gm_1m_2}{2r}$$

$$\frac{E}{E_A}=\frac{v_A}{v_B}=\frac{1.4}{1}$$

Q.45 (3)

$$T\propto r^{3/2}\left[\omega=\frac{2\pi}{T}\right]$$

$$\omega\propto\frac{1}{r^{3/2}}$$

$$\left(\frac{\omega}{2\omega}\right)=\left(\frac{R_1}{r}\right)^{3/2}$$

$$\frac{R_1}{r}=\frac{1}{(2)^{2/3}}$$

$$R_1=\frac{r}{(4)^{1/3}}$$

Q.46 (2)

$$\frac{GMm}{r^2}=\frac{mv^2}{r}$$

Q.47 (3)

$$v=\sqrt{\frac{Gm}{r}}=\sqrt{\frac{Gm}{R+h}}$$

$$v_1=\sqrt{\frac{Gm}{R+\frac{R}{2}}}=\sqrt{\frac{2}{3}}\,v$$

Q.48 (1)

$$v\propto\sqrt{\frac{1}{r}}$$

$$\frac{v_A}{v_B}=\sqrt{\frac{r_B+R}{r_A+R}}$$

Q.49 (3)
Kepler's second law

$$\frac{dA}{dt}=\frac{L}{2M}=\text{constant}$$

Q.50 (1)

$$\frac{A}{T}=\frac{L}{2m}$$

$$L=\frac{2mA}{T}$$

Q.51 (2)

$$F=\frac{GM_sM_e}{R^2}$$

$$F'=\frac{GM_sM_e}{(3R)^2}=\frac{F}{9}$$

Q.52 (3)

$$r\propto T^{2/3}$$

$$\frac{r_1}{r_2}=\left(\frac{3}{24}\right)^{2/3}=\frac{1}{4}$$

$$T=\frac{2\pi r}{v}\Rightarrow v=\frac{2\pi r}{T}$$

$$\frac{v_1}{v_2}=\left(\frac{r_1}{r_2}\right)\left(\frac{T_1}{T_2}\right)=\left(\frac{1}{4}\right)\left(\frac{24}{3}\right)=\frac{2}{1}$$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (B)
Gravitational field at ‘m’ due to hollowed - out lead sphere
= { Field due to solid spere } – { Field due to mass that was removed }

Field due to solid sphere = $\frac{GM}{d^2}$ = E_1 = $\frac{GM}{4R^2}$

Field due to removed mass = $\frac{GM'}{x^2}$ = E_2

$$M' = \frac{M}{\frac{4}{3}\pi R^3}\times\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

And $x = d-\frac{R}{2}$

So, $E_2 = \dfrac{GM}{8\left(d-\dfrac{R}{2}\right)^2} = \dfrac{GM}{8\left(\dfrac{3R}{2}\right)^2} = \dfrac{GM}{18R^2}$

$E_{net} = E_1 - E_2$

$= \dfrac{GM}{R^2}\left\{\dfrac{1}{4}-\dfrac{1}{18}\right\} = \dfrac{7GM}{36R^2}$

$F_{net} = mE_{net} = \dfrac{7GMm}{36R^2}$ **Ans.**

Q.2 (A)

Let's take strip of lenght 'dx' at lenght x, from (0, 0).

Its mass = dm = ρdx = $(a + bx^2)$ dx है।

Force due to this strip on

'm' $= dF = \dfrac{Gm}{x^2}\,dm = Gm\,\dfrac{a+bx^2}{x^2}\,dx$

Total force $F = \int dF = \int_{\alpha}^{\alpha+\ell} Gm\,\dfrac{a+bx^2}{x^2}\,dx$

$= Gm\int_{\alpha}^{\alpha+\ell}\left(\dfrac{a}{x^2}+b\right)dx$

$= Gm\left\{\dfrac{a}{\alpha}-\dfrac{a}{\alpha+\ell}+b\ell\right\}$

$= Gm\left\{a\left(\dfrac{1}{\alpha}-\dfrac{1}{\alpha+\ell}\right)+b\ell\right\}$ **Ans.**

Q.3 (C)

If we take complete spherical shell than gravitational field intensity at P will be zero hence for the hemi spherical shell shown the intensity at P will be along c.

Q.4 (A)

$F_1 = \dfrac{GMm}{R^2}$

$F_2 = \dfrac{GMm}{3R^2}$

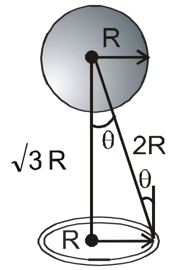
$\therefore$ Change $= \dfrac{2}{3}\,\dfrac{GMm}{R^2}$

Q.5 (A)

In (a) & (b) $\nearrow \dfrac{2GM^2}{a^2}$

In (c) & (d) $\nearrow \dfrac{4GM^2}{a^2}$

Q.6 (D)



$F = \dfrac{GMm}{(2R^2)}\cos\theta = \dfrac{GMm}{(2R^2)}\times\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}GMm}{8R^2}$

Q.7 (C)

From E.C.

$$O = -\dfrac{Gm_1m_2}{d}+\dfrac{1}{2}m_1v_1^2+\dfrac{1}{2}m_2v_2^2$$

from M.C. $m_1v_1 - m_2v_2 = 0$

Now, $\dfrac{Gm_1m_2}{d} = \dfrac{1}{2}m_1v_1^2+\dfrac{1}{2}m_2\left(\dfrac{m_1v_1}{m_2}\right)^2$

$v_1 = \sqrt{\dfrac{2Gm_2^2}{d\,(m_1+m_2)}}$

$v_2 = \sqrt{\dfrac{2Gm_1^2}{d\,(m_1+m_2)}}$

Relative velocity of approach = $v_1 + v_2$

$v = \sqrt{\dfrac{2G\,(m_1+m_2)}{d}}$

Q.8 (A)

$G - \omega^2R = g/2$

$\omega^2R = g/2$

$\dfrac{v^2}{R} = g/2$

$2v^2 = gR$

$v_{es} = \sqrt{2gR}$

$v_{es} = \sqrt{2(2v^2)} = 2v$

Q.9 (B)

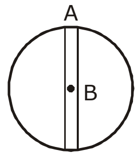
$v_e = \sqrt{2gR} = \sqrt{\dfrac{2GM}{R}}$

from E.C.

$-\dfrac{GMm}{R} = \dfrac{-3GMm}{2R}+\dfrac{1}{2}mv^2$

$\dfrac{GM}{2R} = \dfrac{1}{2}v^2$

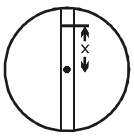
$$v = \frac{v_e}{\sqrt{2}}$$



Q.10 (B)

$$F = \frac{GMmx}{R^3}$$

$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$



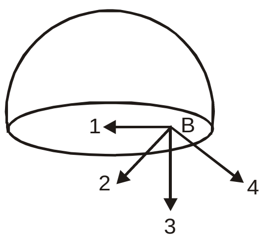
T = 84.6 min

time to each other end $t = \frac{T}{2} = 42.3\text{min}$

Q.11 (D)

Let the possible direction of gravitational field at point B be shown by 1, 2, 3 and 4 (Figure 1). Rotate the figure upside down. It will be as shown in figure 2.

Figure 1



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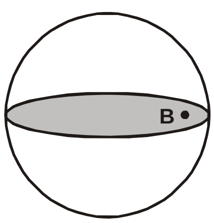


Figure 3

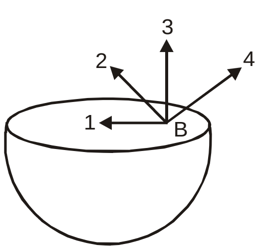
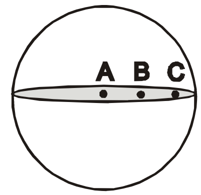


Figure 2

Now on placing upper half of figure 1 on the lower half of figure 2 we get complete sphere. Gravitational field at point B must be zero, which is only possible if the gravitational field is along direction 3. Hence gravitational field at all points on circular base of hemisphere is normal to plane of circular base.

$\therefore$ Circular base of hemisphere is an equipotential surface.

Aliter : Consider a shaded circle which divides a uniformly thin spherical shell into two equal halves.The potential at points A,B and C lying on the shaded circle is same. The potential at all these points due to upper hemisphere is half that due to complete sphere.Hence potential at points A,B and Cis also same due to upper hemispehre



Q.12 (C)

$$\frac{\frac{Gm_1}{r_1}}{\frac{Gm_2}{r_2}} = \frac{3}{4}$$

$$\frac{m_1}{4\pi r_1^2} = \frac{m_2}{4\pi r_2^2}$$

$m_1 + m_2 = m$

$$\frac{m}{4\pi R^2} = \frac{m_1}{4\pi r_1^2}$$

or $$= \frac{Gm}{R \times \frac{Gm_1}{r_1}} = \frac{5}{3}$$ **Ans.**

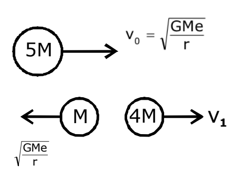
Q.13 (D)

$$\frac{-GM_e m}{R} = E_{initial}$$

$$E_{final} = \frac{-GM_e m}{2 \times 2R}$$

$\therefore$ difference $= -\frac{GM_e m}{R} + \frac{GM_e m}{4R} = \frac{3}{4}$ mgR

Q.14 (B)



$5Mv_0 = -Mv_0 + 4Mv_1$

$$\frac{3v_0}{2} = v_1$$

$$v_1 = \frac{3}{2}\sqrt{\frac{GM_e}{r}}$$

$$T.E. = -\frac{GM_e 4m}{r} + \frac{1}{2}4m\left(\frac{9}{4}\frac{GM_e}{r}\right)$$

T.E. > 0

Q.15 (A)

$$\frac{dA}{dt} = \frac{m\sqrt{\frac{GMe}{R}}\ .\ R}{2m}$$

$$\frac{dA}{dt} \propto \sqrt{R}$$

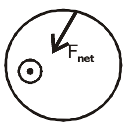
Q.16 (A)

$$\frac{-GM_e m}{r} = P.E.$$

$$v = \sqrt{\frac{GM_e}{r}}$$

$$\omega = \frac{2\pi}{T} \qquad r\uparrow\ T\uparrow\ \omega\downarrow$$

Q.17 (A)

(A) 

(B) Both direction and Magnitude not change

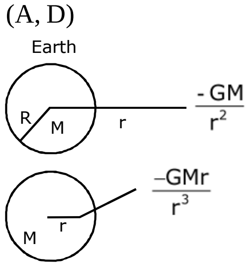
(C) Total Mechanical is constant

(D) Linear momentum changes becoz v change as r changes but rv = constant

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

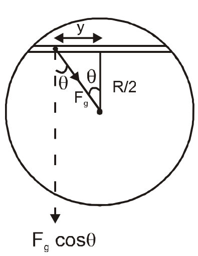
Q.1 (A, D)



Q.2 (B, C)

$$F_g = \frac{GMmr}{R^3}$$

$$\text{pressing force} = F_g \cos\theta = \frac{GMmr\cos\theta}{R^3}$$



$$= \frac{GMm}{2R^2} = \text{constant.}$$

$$a = \frac{Fg\sin\theta}{m}$$

$$= \frac{GMr\sin\theta}{R^3}$$

$$a = \frac{GMy}{R^3}$$

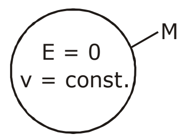
Q.3 (A, D)

$m_1 \longrightarrow F \qquad \overset{F}{\longleftarrow} m_2$

$$a_1 = \frac{F}{m_1} \quad a_2 = \frac{F}{m_2}$$

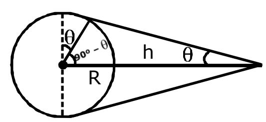
$m_1 < m_2 \Rightarrow a_1 > a_2$

Q.4 (B, C, D)



Q.5 (A, C)

$$\sin\theta = \frac{R}{R+h}$$



Area covered = $R^2 2\pi$ (1 – cos(90–θ))
= $2\pi R^2$ (1 – sinθ)

Area escaped = $4\pi R^2 - 2\pi R^2$ (1 – sinθ)
= $2\pi R^2$ (1 + sinθ)

Q.6 (A, B, C, D)

The angular velocity of the geostationary satellite must be equal to angular velocity of earth in both direction and magnitude.

Q.7 (A, C, D)

inside a uniform spherical shell

$E_{in} = 0$

$$V_{in} = \text{constant} = \frac{Gm}{R}$$

Q.8 (A, C)

$$\text{Escape velocity} = \sqrt{\frac{2GM}{R}} = V_e$$

$$\text{Orbital velocity} = \sqrt{\frac{GM}{R}} = V_0$$

Escape velocity = $\sqrt{2}$ × orbital velocity → (A)

$$\frac{1}{2}\,mV_e^2 = 2\times\frac{1}{2}\,mV_0^2 \rightarrow (C)$$

Q.9 (B, C)

$\omega_S = \dfrac{2\pi}{1.5}$,

$\omega_E = \dfrac{2\pi}{24}$

$\omega_{\text{west to east}} = 2\pi\left[\dfrac{1}{1.5}-\dfrac{1}{24}\right]$

$T_{\text{west to east}} = \dfrac{2\pi}{\omega_{\text{west to east}}} = 1.6$ hours

Similarly

$\omega_{\text{east to west}} = 2\pi\left[\dfrac{1}{1.5}+\dfrac{1}{24}\right]$

$T_{\text{east to west}} = \dfrac{24}{17}$ hours

Q.10 (B, C)

$PE = -G\,m_1 m_2/r$, $ME = -G\,m_1 m_2/2r$

On decreasing the radius of orbit PE and ME decreases

Q.11 (A, D)

In case of earth the gravitational field is zero at infinity as well as the the centre and the potential is minimum at the centre.

Q.12 (A, C)

Radius decreases $\Rightarrow$ Velocity increases due to which K.E. increases

$m.v.r \Rightarrow m\sqrt{\dfrac{GM}{r}}\,.\,r \Rightarrow \alpha\sqrt{r}$

$T^2 \propto r^3$

Q.13 (B, D)

Theory

Q.14 (A, B, C)

$v = \sqrt{\dfrac{GM_e}{R}}$ (Max.)

$T^2 \propto r^3 \quad r\downarrow \quad T\downarrow$

T.E. $= \dfrac{-GMm}{2R}$

Q.15 (A, D)

$T = \dfrac{2\pi}{\sqrt{GM_e}}\,r^{3/2}$

$g = \dfrac{GM_e}{R_e^2}$ and $T_{min.}$ at $r_{min.} = R$

$\Rightarrow \quad T_{min.} = \dfrac{2\pi R^{3/2}}{\sqrt{gR_e^2}} = 2\pi\sqrt{\dfrac{R}{g}}$

Q.16 (C)

Q.17 (A)

Q.18 (A)

$T^2 = \left(\dfrac{4\pi^2}{GM}\right)R^3$

$R = \left(\dfrac{GM}{4\pi^2}\right)^{1/3} T^{2/3}$

$\log R = \dfrac{2}{3}\log T + \dfrac{1}{3}\log\left(\dfrac{GM}{4\pi^2}\right)$

$y = mx + c$

(3) Slope $= m = \dfrac{2}{3}$

intercept $c = \dfrac{1}{3}\log\left(\dfrac{GM}{4\pi^2}\right) = 6$

$\log\dfrac{\left(\dfrac{20}{3}\times10^{-11}\right)M}{4\times10} = 18$

(4) $M = 6\times10^{29}$ Kg

(5) $T^2 \propto R^3$

$\left(\dfrac{R_A}{R_B}\right)^3 = \left(\dfrac{R_A}{R_B}\right)^2 = \left(\dfrac{\omega_B}{\omega_A}\right)^2$

$\left(\dfrac{R}{4R}\right)^3 = \left(\dfrac{\omega_B}{\omega_A}\right)^2 \Rightarrow \left(\dfrac{\omega_B}{\omega_A}\right) = \left(\dfrac{1}{8}\right)$

$\omega_{rel} = 8\omega_0 - \omega_0 = 7\omega_0$

$\theta_{rel} = (\omega_{rel})\,t$

$2\pi = (T\omega_0)\,t$

$t = \dfrac{2\pi}{T\omega_0}$

Q.19 (B)

Let M and R be the mass and radius of the earth respectively. If m be the mass of satellite, then escape velocity from earth $\nu_e = \sqrt{(2\,Rg)}$

Velocity of satellite $\nu_s = \frac{\nu_e}{2} = \sqrt{(2Rg)}/2$ .... (1)

Further, $\nu_s = \sqrt{\left(\frac{GM}{r}\right)} = \sqrt{\left(\frac{R^2g}{R+h}\right)}$

$\therefore \quad \nu_s^2 = \frac{R^2g}{R+h}$

h = R = 6400 km

Q.20 (C)

$T^2 = \frac{4\pi^2}{Gm}x^3$

Hence time period of revolution T is

$T = 2\pi\sqrt{\frac{x^3}{Gm}}$ (Put x = 2R)

$\therefore \quad T = 2\pi\sqrt{\frac{8R}{g}}$

Q.21 (A)

Now total energy at height h = total energy at earth's surface (from principle of conservation of energy)

$\therefore \quad 0 - GM\frac{m}{R+h} = \frac{1}{2}mv^2 - GM\frac{m}{R}$

or $\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R}$

($\because$ h = R)

$\therefore \quad v = \sqrt{gR}$

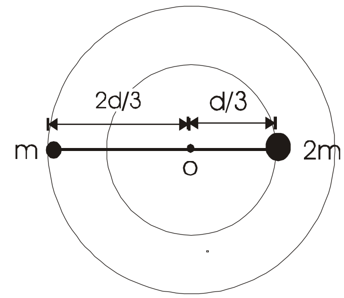
Q.22 (D)

Q.23 (B)

Q.24 (B)

(22 to 24)

Let the angular speed of revolution of both stars be ω about the common centre , that is, centre of mass of system.



The centripetal force on star of mass m is

$m\omega^2\frac{2d}{3} = \frac{Gm(2m)}{d^2}$. Solving we get

$T = \sqrt{\frac{4\pi^2}{3Gm}d^3}$

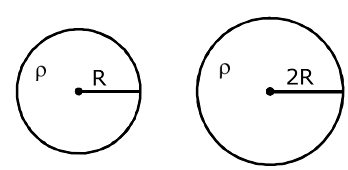
The ratio of angular momentum is simply the ratio of moment of inertia about center of mass of system.

$$\frac{L_m}{L_M} = \frac{I_m\omega}{I_M\omega} = \frac{m\left(\frac{2d}{3}\right)^2}{2m\left(\frac{d}{3}\right)^2} = 2$$

Similarly, The ratio of kinetic energy is simply the ratio of moment of inertia about center of mass of system.

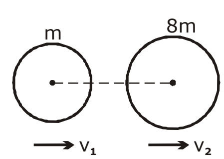
$$\frac{K_m}{K_M} = \frac{\frac{1}{2}I_m\omega^2}{\frac{1}{2}I_M\omega^2} = \frac{m\left(\frac{2d}{3}\right)^2}{2m\left(\frac{d}{3}\right)^2} = 2$$

Q.25 (B)



$\rho\,\frac{4}{3}\pi R^3 = m$ $\qquad \rho\,\frac{4}{3}(2R^3) = \mathbf{8}m$

from E.C.



$O + O = -\frac{Gm8m}{3R}$

$$+\frac{1}{2}mv_1^2+\frac{1}{2}8mv_2^2 \qquad ....(1)$$

from M.C.

$mv_1 = 8mv_2$

$v_1 = 8v_2 \qquad ....(2)$

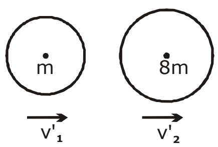
Put value from eq. (2) to eq. (1)

$$\frac{G8m^2}{3R}=\frac{1}{2}m64v_2^2+\frac{1}{2}8m_2^2$$

$$v_2^2=\frac{2Gm}{27R}$$

$$\Rightarrow \quad v_1^2=\frac{64\times 2Gm}{27R}$$

After collision



$u_1 = v_1$

$u_2 = -v_2$

$\therefore$

$$v'_1=\frac{o+\frac{8m}{2}(-v_2-v_1)}{9m}=\frac{-4}{9}(v_1+v_2)$$

$$v'_2=\frac{v_1+v_2}{18m}$$

Initial K.E. $=\frac{Gm8m}{3R}$

final K.E. $=\frac{1}{2}mv_1^2+\frac{1}{2}8mv_2^2$

$$=\frac{1}{2}m\,\frac{16}{81}(v_1+v_2)^2+\frac{1}{2}8m(v_2+v_1)^2\,\frac{1}{(18)^2}$$

$$=\frac{m}{g}(v_1+v_2)^2=\frac{2}{3}\frac{Gm^2}{R}$$

Q.26 (A)

After collision r is max. separation

from M.C.

$8mv'_2 + mv'_1 = 9mv$

$$8m\left[\frac{v_1+v_2}{18}\right]-m\left[\frac{4}{9}(v_1+v_2)\right]=9mv$$

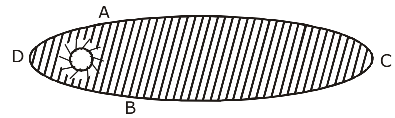
$v = 0$

from E.C.

$$\frac{-G8m^2}{r}+\frac{1}{2}8mv^2+\frac{1}{2}mv^2=\frac{2}{3}\frac{Gm^2}{R}-\frac{G8m^2}{3R}$$

$\Rightarrow r = 4R$

Q.27 (B)



$t_1 > t_2$ between area is ACB is greater than ADB.

Q.28 (C)

Both D & C between

Total energy is always –ve

Q.29

I	II	III
A	p	w
B	r	u
C	q	v
D	p	t

P.E. $=-\frac{GMm}{r} \Rightarrow$ K.E. $=\frac{1}{2}mV^2$

Total energy $=-\frac{GMm}{r}+\frac{1}{2}mV^2$

T.E. = 0 if $-\frac{GMm}{r}+\frac{1}{2}mV^2=0 \Rightarrow v=\sqrt{\frac{2GM}{r}}$

For $v<\sqrt{\frac{2GM}{r}}$, T.E. is – ve

for $v>\sqrt{\frac{2GM}{r}}$, T.E. is + ve

If V is $\sqrt{\frac{GM}{r}}$ i.e. equal to orbital velocity, path is circular.

If T.E. is negative, path is elliptical.

If T.E. is zero, path is parabolic.

If T.E. is positive, path is hyperbolic.

Q.30 (A) p,r (B) p,r (C) q,r (D) p,r

(A) At centre of thin spherical shell $V \neq 0$, $E = 0$.

(B) At centre of solid sphere $V \neq 0$, $E = 0$.

(C) At centre of spherical cavity inside solid sphere $V \neq 0$, $E \neq 0$.

(D) At centre of two point masses $V \neq 0$, $E=0$.

NUMERICAL VALUE BASED

Q.1 [3]

w.r.t. COM K.E. $=\frac{1}{2}$ (red mass) v_{rel}^2

w.r.t. COM Angular momentum $=\frac{mr}{2}v_{rel}$

$\therefore$ Equating energy

$$\frac{1}{2}\frac{m}{2}v_0^2 - \frac{Gm^2}{r^0} = \frac{1}{2}\frac{m}{2}v_{rel}^2 - \frac{Gm^2}{r}$$

(Here v_{rel} is relative velocity $\perp$ to line as v_{rel} along the line joining is zero when separation is either min. or max.)

Angular momentum conservation

$$\frac{mr_0}{2}v_0 = \frac{mr}{2}v_{rel}$$

$$\therefore \quad v_{rel} = \frac{r_0 v_0}{r}$$

solving $3r^2 - 4rr_0 + r_0^2 = 0$

$$\therefore r_{max} = r_0 \qquad r_{min} = \frac{r_0}{3}$$

ratio = 3 (ans)

Q.2 [3000 sec]

$$T = \frac{2\pi R}{V}$$

$$= 2\pi\sqrt{\frac{R^3}{GM}} = \sqrt{\frac{3\pi}{GP}}$$

$= 3 \times 10^3$ sec = 3000 sec.

Q.3 [8]

$$\frac{-GM \times 3M}{d} + \frac{1}{2}\mu V_{rel}^2 = 0$$

$$\frac{+3GM^2}{d} = \frac{1}{2} \times \frac{3M^2}{4M} \times V_{rel}^2$$

$$\Rightarrow V_{rel}^2 = \frac{8GM}{d}$$

$$V_{rel} = \sqrt{\frac{8GM}{d}} \Rightarrow \eta = 8 \text{ Ans.}$$

Q.4 [2]

$$dF = \frac{Gmdm}{x^2} = \frac{Gm}{x^2}\left(\frac{K}{x}\right)dx$$

Integrate from d to ∞

$$F = \frac{GmK}{2d^2}$$

Q.5 [23]

$$T = \frac{2\pi(a)^{3/2}}{\sqrt{GM}} \text{ so } M = \frac{4\pi^2(a)^3}{GT^2}$$

Putting values we get

$M = 2 \times 10^{21}$ kg

Q.6 [4 J]

$W_{ext} + W_g = 4K = 0$

$W_{ext} - m_4 V = 0$

$$Wext = 2 \times \frac{4}{2} = 4 \text{ J}$$

Q.7 [5]

$m\,ur = m\,(v_0 + v)\,a$

$$\Rightarrow u = \sqrt{\frac{5GM_e a}{4r^2}} \text{ as } v_0 = \sqrt{\frac{GM_e}{a}}$$

$$\text{also } \frac{1}{2}mu^2 - \frac{GM_e m}{r} = \frac{1}{2}m\,(v_0 + v)^2 - \frac{GM_e m}{a}$$

Q.8 [2]

$$V_e = \sqrt{\frac{2GM}{R}} \quad \Rightarrow V_e = \sqrt{\frac{2G.\frac{4}{3}\pi R^3 \rho}{R}}$$

$$\Rightarrow V_e = \sqrt{8G\pi\rho}R$$

$$\frac{V_A}{V_B} = \frac{R_A}{R_B} \quad \Rightarrow \quad \frac{V_A}{V_B} = \frac{2}{1}$$

Q.9 [12 hrs]

$$T = \frac{2\pi r}{\omega_s + \omega_e}$$

$$\Rightarrow \quad T = \frac{2\pi r}{2\omega_e} \quad \Rightarrow \quad T = \frac{24}{2}$$

$\Rightarrow \quad T = 12h$

Q.10 [40]

$$\frac{dA}{dt} = \frac{r^2\omega}{2} \text{ is constant}$$

$$\therefore \frac{dA}{dt} = \frac{r_{max}^2\omega_{min}}{2} = \frac{r_{min}^2\omega_{max}}{2}$$

$$\Rightarrow \quad \omega_{min} = \frac{2dA/dt}{r_{max}^2}$$

$$V_{max} = \omega_{min} r_{min} = \frac{2dA/dt}{r_{min}} = 40 \text{ k m/s}$$

KVPY
PREVIOUS YEAR'S

Q.1 (D)

$$B.E = \frac{GmM}{2R}$$

$$B.E = \frac{1}{2}mv_0^2$$

so, $\alpha = 1$

Q.2 (A)

Using conservation of angluar momentum

$V_P . r_P = V_A . r_A$

$$\frac{V_P}{V_A} = \frac{r_A}{r_P} = \frac{a+ae}{a-ae}$$

Q.3 (D)

$$g = \frac{GM}{(R+h)^2}$$

h << R

$$g \approx \frac{GM}{(R^2)} \text{ towards the earth}$$

Q.4 (D)

At earth surface acceleration due to gravity $g = \frac{GM}{R^2}$

At height = 9000m, Radius of orbit of ball is 6400 + 9km $\Rightarrow$ radius r > R

Radius is almost equal to radius of earth.

(v) orbital velocity of ball = $\sqrt{\frac{GM}{r}}$

Acceleration = $\frac{v^2}{r} \Rightarrow \frac{GM}{r^2}$

as r is very near to R

$\therefore$ Acceleration $= \frac{GM}{R^2} = g$

Q.5 (A)

Planet sun system is bounded system

$\therefore$ Total energy of the system is negative

TE = KE + PE

$\Rightarrow$ K – | U | {PE is negative here}

as TE is negative

$\Rightarrow$ | U | > K

Q.6 (A)

Using energy conservation

$$-\frac{GMM}{R} + \frac{1}{2}mV^2 = -\frac{GMM}{5R}$$

$$V = 2\sqrt{\frac{2GM}{5R}}$$

V is the velocity by which object is projected. When object return to earth its speed will be V.

Q.7 (A)

$V(r) = Kr^{-n}$

gravitational field = E = $-\frac{dV}{dr}$

$$= (-K)\frac{d}{dr}\left(r^{-n}\right) = (-K)\,(-n)r^{-n-1} = \frac{Kn}{r^{n+1}}$$

force on mass = E × M, where M = mass of body

$$\therefore \quad ME_1 = \frac{Mv_1^{\,2}}{r_1} \qquad ME_2 = \frac{Mv_2^{\,2}}{r_2}$$

$$\therefore \quad \frac{v_1^{\,2}}{v_2^{\,2}} = \frac{r_1E_1}{r_2E_2}$$

$$\Rightarrow \frac{v_1^{\,2}}{v_2^{\,2}} = \frac{r_1}{r_2}\frac{Kn}{r_1^{\,n+1}}\frac{r_2^{\,n+1}}{Kn} \qquad \Rightarrow \frac{v_1^{\,2}}{v_2^{\,2}} = \frac{r_2^{\,n}}{r_1^{\,n}} \Rightarrow v_1^{\,2}\, r_1^{\,n} = v_2^{\,2}\, r_2^{\,n}$$

Q.8 (C)

$$T^2 = \frac{4\pi^2}{GM}a^2$$

$$\frac{4\pi^2}{GM} = 1$$

T = in year

a = radius in A.U.

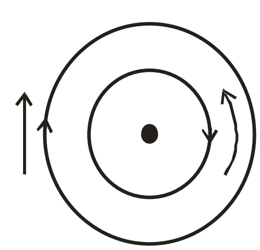
$\therefore$ T = 3 days = $\frac{3}{365}$ year

$$\therefore a = \left(\frac{3}{365}\right)^{2/3}$$

a = 0.04 A.U.

Q.9 (D)

$T^2 \propto r^3$



$$\frac{9}{24^2} = \left(\frac{3\times10^4}{r^3}\right)^3$$

$$\frac{3\times3}{24\times24} = \left(\frac{3\times10^4}{r^3}\right)^3$$

$$\frac{1}{4} = \frac{3\times10^4}{r}$$

$r = 12 \times 10^4$

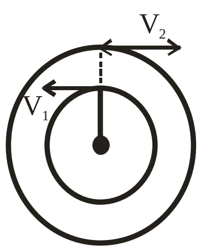
Orbital speed of S_2 seen form planet = $\omega_2 r$

$$= \frac{2\pi}{24}\times12\times10^4$$

$= \pi \times 10^4$ km h^{-1}

$$V_1 = \omega_1 r_1 = \frac{2\pi}{3}\times3\times10^4$$

$\Rightarrow$ $2\pi \times 10^4$ km h^{-1}



$$\left(\frac{2\pi}{3}+\frac{2\pi}{24}\right)t = \pi$$

$$\frac{9t}{24} = \frac{1}{2}$$

$$t = \frac{12}{9}\text{hr}$$

Anlge rotate by both satellite

$$\theta_1 = \frac{2\pi}{3}\times\frac{12}{9} \Rightarrow \frac{8\pi}{9}$$

$$\theta_2 = \frac{2\pi}{24}\times\frac{12}{9} \Rightarrow \frac{\pi}{9}$$

Velocity of S_2 seen from

$S_1 = V_1 + V_2$

$= 3\pi \times 10^4$ km h^{-1}

Q.10 (D)

A lunar eclipse can occur on a new moon and full moon day.

Q.11 (C)

Gravitational force = $F_G = \dfrac{Gm_1m_2}{r^2}$

Electrostatic force = $F_e = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r^2}$

$$\frac{F_G}{F_e} = \frac{Gm_1m_2\cdot4\pi\epsilon_0}{q_1q_2}$$

$$= \frac{6.7\times10^{-11}\times9.1\times10^{-31}\times9.1\times10^{-31}}{1.6\times10^{-19}\times1.6\times10^{-19}\times9\times10^{9}}$$

$= 24 \times 10^{-44}$

Q.12 (D)

At a height 'h'

$$g_h = g_0\left(1-\frac{2h}{R}\right)$$

when h (10 km) < R (6400 km)

at a depth 'd'

$$g_d = g_0\left(1-\frac{d}{R}\right)$$

here $g_0 = \dfrac{GM}{R^2}$

Now $g_h = g_d$

$\Rightarrow$ d = 2h

d = 20 km

JEE-MAIN PREVIOUS YEAR'S

Q.1 (1)

$$T = 2\pi\sqrt{\frac{\ell}{g_{planet}}} \Rightarrow 2 = 2\pi\sqrt{\frac{2}{g_{planet}}}$$

$$\Rightarrow \frac{1}{\pi^2} = \frac{2}{g_{planet}} \Rightarrow g_{planet} = 2\pi^2 \text{ m/sec}^2$$

Q.2 (2)

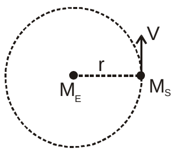
$$g_1 = \frac{GM}{9R^2}$$

$$g_2 = \frac{GM}{9R^2} - \frac{G\left(M/8\right)}{\left(\frac{5R}{2}\right)^2}$$

$$= \frac{GM}{9R^2} - \frac{GM}{R^2}\left(\frac{1}{50}\right) \qquad = \frac{41}{50\times9}\left(\frac{GM}{R^2}\right)$$

$$\therefore \frac{g_1}{g_2} = \frac{50}{41}$$

Q.3 (1)



$$v = \sqrt{\frac{2GM_e}{r}}$$

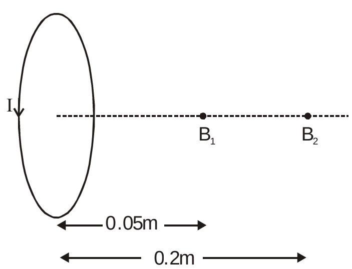
$$T = \frac{2\pi r}{\sqrt{\frac{2GM_e}{r}}} = 2\pi r\sqrt{\frac{r}{2GM_e}}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{2GM_e}} = \sqrt{\frac{2\pi^2 r^3}{GM_e}}$$

$$T_2 - T_1 = \sqrt{\frac{2\pi^2(8000\times10^3)^3}{G\times6\times10^{24}}} - \sqrt{\frac{2\pi^2(7000\times10^3)^3}{G\times6\times10^{24}}}$$

$= 2887.15$ sec

Q.4 (1)



$$B = \frac{\mu_0 NIR^2}{2(R^2+x^2)^{3/2}} \Rightarrow \frac{B_1}{B_2} = \frac{8}{1} = \frac{(R^2+x_2^2)^{3/2}}{(R^2+x_1^2)^{3/2}}$$

$$\left(\frac{R^2+x_2^2}{R^2+x_1^2}\right)^3 = 64 \Rightarrow \frac{R^2+x_2^2}{R^2+x_1^2} = 4$$

$$R^2 + x_2^2 = 4R^2 + 4x_1^2$$

$$3R^2 + x_2^2 = 4x_1^2$$

$$= \left(\frac{2}{10}\right)^2 - 4\left(\frac{5}{100}\right)^2$$

$$= \frac{4}{100} - \frac{1}{100}$$

$$3R^2 = \frac{1}{10}$$

$$R = \frac{1}{10}$$

$R = 0.1$ m

Q.5 (3)

Theoretical.

Q.6 (10)

$$\frac{GMm}{R} + \frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$\frac{1}{2}mv^2 = \frac{10GMm}{11R}$$

$$v = V_{escape}\, x\sqrt{\frac{10}{11}}$$

$x = 10$

Q.7 (3)

$$F = \frac{GMm\sqrt{8}R}{(R^2+8R^2)^{3/2}}$$

$$= \frac{GMm}{R^2}\times\frac{\sqrt{8}}{27}$$

Q.8 (1)

Q.9 (4)

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GMr}{R^3}$$

$$OA = \frac{4R}{9} = r$$

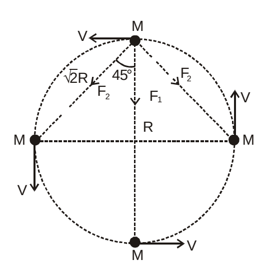
$$AB = R - \frac{4R}{9} = \frac{5R}{9}$$

$OA : AB$

$$\frac{4R}{9} : \frac{5R}{9} \Rightarrow 4:5 = x:y$$

$(x = 4)$

Q.10 (2)



Net force on one particle

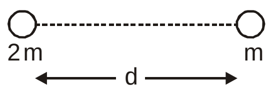
$F_{net} = F_1 + 2F_2 \cos 45^o$ = Centripetal force

$$\Rightarrow \frac{GM^2}{(2R)^2} + \left[\frac{2GM^2}{\left(\sqrt{2}R\right)^2}\cos 45^O\right] = \frac{MV^2}{R}$$

$$V = \frac{1}{2}\sqrt{\frac{GM}{R} + (1 + 2\sqrt{2})}$$

$$V = \frac{1}{2}\sqrt{G(1 + 2\sqrt{2})}$$

Q.11 (1)



$$\frac{G(m)(2m)}{d^2} = m\omega^2 \times \frac{2d}{3}$$

$$\frac{2Gm}{d^2} = m\omega^2 \times \frac{2d}{3}$$

$$\omega^2 = \frac{3Gm}{d^3}$$

$$\omega = \sqrt{\frac{3Gm}{d^3}}\ ; \qquad T = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

Q.12 (1)

mg = 49

$m(g - \omega^2 R)$ will be less than mg.

Q.13 (3)

By angular momentum conservation :

$mv_1r_1 = mv_2r_2$

$$v_1 \frac{48\times10^{14}}{1.6\times10^{12}} = 3000\text{m / sec}$$

Q.14 (3)

Energy given = $U_f - U_i$

$$= 0 - \left(-\frac{3}{5}\frac{GM^2}{R}\right)$$

$$= \frac{3}{5}\frac{GM^2}{R}$$

x = 3

Q.15 (64)

$$V_e = \sqrt{\frac{2Gm}{R}} \qquad \text{.... (1)}$$

$$10V_e = \sqrt{\frac{2Gm}{R'}} \qquad \text{.... (2)}$$

$$\therefore\ 10 = \sqrt{\frac{R}{R'}}$$

$$\Rightarrow R' = \frac{R}{100} = \frac{6400}{100} = 64 \text{ km}$$

Q.16 (3)

$T \propto R^{3/2}$

$$\frac{24}{T} = \left(\frac{12R}{3R}\right)^{3/2} \Rightarrow T = 3\text{hr}$$

Q.17 (2)

$T^2 \propto R^3$

$$\left(\frac{T'}{T}\right)^2 = \left(\frac{9R}{R}\right)^3$$

$T'^2 = T^2 \times 9^3$

$T' = T \times 3^3$

$T' = 27\ T$

Q.18 (4)

For objects to float

$mg = m\omega^2 R$

ω = angular velocity of earth.

R = Radius of earth

$$\omega = \sqrt{\frac{g}{R}} \qquad \text{... (1)}$$

Duration of day = T

$$T = \frac{2\pi}{\omega} \qquad \text{... (2)}$$

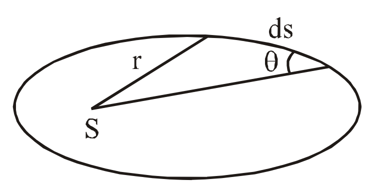
$$\Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$

$$= 2\pi\sqrt{\frac{6400\times10^3}{10}}$$

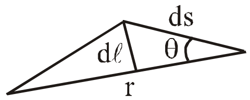
$$\Rightarrow = \frac{T}{60} = 83.775 \text{ minutes}$$

$\simeq$ 84 minutes

Q.19 (4)



For small displacement ds of the planet its area can be written as



$$dA = \frac{1}{2} r d\ell$$

$$= \frac{1}{2} \text{ r ds sin } \theta$$

$$\text{A.vel} = \frac{dA}{dt} = \frac{1}{2} r \sin\theta \frac{ds}{dt} = \frac{Vr\sin\theta}{2}$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{mVr\sin\theta}{m} = \frac{L}{2m}$$

Q.20 (1)

Q.21 (4)

Q.22 (500)

Q.23 (4)

Q.24 (4)

Q.25 (3)

Q.26 (4)

Q.27 (2)

Q.28 (4)

Q.29 (4)

Q.30 (1)
Inside a spherical shell, gravitational field is zero and hence potential remains same everywhere
Hence option (1)

Q.31 (2)

Q.32 (4)
To convert pulsating dc into steady dc both of mentioned method are correct.

$$g_{up} = \frac{g}{\left(1 + \frac{r}{R}\right)^2}$$

$$g_{down} = g\left(1 - \frac{r}{R}\right)$$

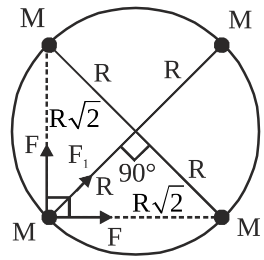
$$\frac{g_{down}}{g_{up}} = \left(1 - \frac{r}{R}\right)\left(1 + \frac{r}{R}\right)^2$$

$$= \left(1 - \frac{r}{R}\right)\left(1 + \frac{2r}{R} + \frac{R^2}{R}\right)$$

$$= 1 + \frac{r}{R} - \frac{r^2}{R^2} - \frac{r^3}{R^3}$$

Q.33 [2]

Q.34 (2)



$$F_{net} = \frac{MV^2}{R}$$

$$\sqrt{2}F + F_1 = \frac{MV^2}{R}$$

$$\sqrt{2}\frac{GMM}{\left(\sqrt{2}R\right)^2} + \frac{GMM}{(2R)^2} = \frac{MV^2}{R}$$

$$\frac{GM}{R}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = V^2$$

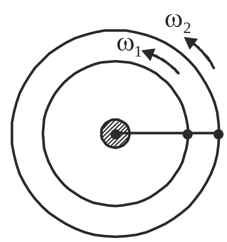
$$\frac{GM}{R}\left(\frac{4+\sqrt{2}}{4\sqrt{2}}\right) = V^2$$

$$V = \sqrt{\frac{GM\left(4+\sqrt{2}\right)}{R4\sqrt{2}}}$$

$$V = \frac{1}{2}\sqrt{\frac{GM\left(2\sqrt{2}+1\right)}{R}}$$

Option (2)

Q.35 (3)



T_1 = 1 hour
$\Rightarrow \omega_1 = 2\pi$ rad/hour
T_2 = 8 hours

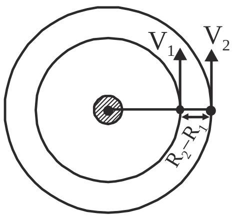
$$\Rightarrow \omega_2 = \frac{\pi}{4} \text{ rad / hour}$$

$R_1 = 2 \times 10^3$ km

As $T^2 \propto R^3$

$$\Rightarrow \left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2$$

$$\Rightarrow \frac{R_2}{R_1} = \left(\frac{8}{1}\right)^{2/3} = 4 \Rightarrow R_2 = 8 \times 10^3 \text{ km}$$



$V_1 = \omega_1 R_1 = 4\pi \times 10^3$ km / h

$V_2 = \omega_2 R_2 = 2\pi \times 10^3$ km / h

$$\text{Relative } \omega = \frac{V_1 - V_2}{R_2 - R_1} = \frac{2\pi \times 10^3}{6 \times 10^3}$$

$$= \frac{\pi}{3} \text{ rad / hour}$$

x = 3

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 (B)

$V_e = \sqrt{2}v_0$

$$KE = \frac{1}{2}mv_e^2 = \frac{1}{2}m\left(\sqrt{2}v_0\right)^2 = mv_0^2$$

Q.2 (B,D)

$$V_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2.G\rho.\frac{4}{3}\pi R^3}{R}} = \sqrt{\frac{4G\rho}{3}}\ R$$

$V_{es} \propto R$

Sarface area of P = A = $4\pi R_P^2$

Surface area of Q = 4A = $4\pi\ R_Q^2$

$\Rightarrow R_Q = 2R_p$

mass R is $M_R = M_P + M_Q$

$$\rho\frac{4}{3}\pi R_R^3 = \rho\frac{4}{3}\pi R_P^3 + \rho\frac{4}{3}\pi R_Q^3$$

$\Rightarrow R_R^3 = R_P^3 + R_Q^3$

$= 9R_P^3$

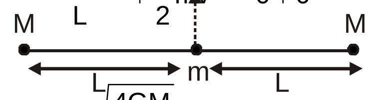
$R_R = 9^{1/3} R_P \Rightarrow R_R > R_Q > R_P$

Therefore $V_R > V_Q > V_P$

$$\frac{V_R}{V_P} = 9^{1/3} \text{ and } \frac{V_P}{V_Q} = \frac{1}{2}$$

Q.3 (B,D)

$$-\frac{GM.2m}{L} + \frac{1}{2}mv^2 = 0 + 0$$



$$\Rightarrow v = \sqrt{\frac{4GM}{L}}$$

Q.4 (B)

Given, $R_{planet} = \dfrac{R_{earth}}{10}$ and

$$\text{density, } \rho = \frac{M_{earth}}{\frac{4}{3}\pi R_{earth}^3} = \frac{M_{Planet}}{\frac{4}{3}R_{planet}^3} \Rightarrow M_{planet}$$

$$= \frac{M_{earth}}{10^3}$$

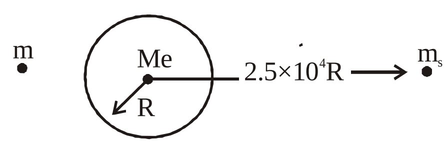
$$g_{\text{surface of planet}} = \frac{GM_{planet}}{R_{planet}^2} = \frac{GM_e.10^2}{10^3.R_e^2} = \frac{GM_e}{10R_e^2} =$$

$$\frac{g_{\text{surface of earth}}}{10}$$

$g_{\text{depth of planet}} = g_{\text{surface of planet}}\left(\dfrac{x}{R}\right)$ where x = distance from centre of planet

$$T = \int_{4R/5}^{R} \lambda\, dx\ g\left(\frac{x}{R}\right) = \frac{\lambda g}{R}\left[\frac{x^2}{2}\right]_{4R/5}^{R} = 108 \text{ N}$$

Q.5 (C)



$$\text{Given } \sqrt{\frac{2GM_e}{R}} = 11.2 \text{km/s}$$

$$\frac{1}{2}mv^2 - \frac{GmM_e}{R} - \frac{GM_s m}{2.5 \times 10^4} \geq 0$$

for $v = v_e$

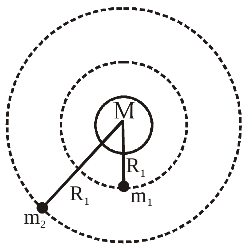
$$v_e^2 = \frac{2GM_e}{R} + \frac{2GM_s}{2.5 \times 10^4 R}$$

$$= \frac{2GM_e}{R} + \frac{6\times10^5 GM_e}{2.5\times10^4 R}$$

$$= \frac{GM_e}{R}(2+24)$$

$$= \sqrt{\frac{26GM_e}{R}} = 40.4\text{km}/\text{sec}.$$

Q.6 (B)



$$\left.\begin{array}{l}\dfrac{m_1}{m_2}=2\\ \dfrac{R_1}{R_2}=4\end{array}\right\}\text{given}$$

$$\frac{GMm_1}{R_1^2} = \frac{m_1v_1^2}{R_1}$$

$$v_1^2 = \frac{GM}{R_1},\ v_2^2 = \frac{GM}{R_2}$$

$$\frac{v_1^2}{v_2^2} = \frac{R_2}{R_1} = 4$$

(P) $\frac{v_1}{v_2} = 2$

(Q) $L = mvR$

$$\frac{L_1}{L_2} = \frac{m_1v_1R_1}{m_2v_2R_2} = 2\times2\frac{1}{4} = 1$$

(R) $K = \frac{1}{2}mv^2$

$$\frac{K_1}{K_2} = \frac{m_1v_1^2}{m_2v_2^2} = 2\times(2)^2 = 8$$

(S) $T = 2\pi R/V$

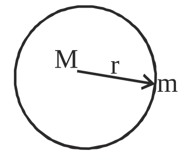
$$\frac{T_1}{T_2} = \frac{R_1}{v_1}\times\frac{v_2}{R_2} = \frac{R_1}{R_2}\times\frac{v_2}{v_1} = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

Q.7 (D)

Let total mass included in a sphere of radius r be M.
For a particle of mass M,

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{GMm}{r} = 2K \quad \Rightarrow M = \frac{2Kr}{GM}$$



$$\therefore dM = \frac{2Kdr}{GM}$$

$$\Rightarrow (4\pi r^2dr)\rho = \frac{2Kdr}{GM} \qquad \Rightarrow \rho = \frac{K}{2\pi r^2GM}$$

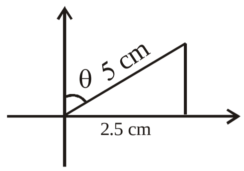
$$\therefore n = \frac{\rho}{m} = \frac{K}{2\pi r^2m^2G}$$

Q.8 (9)

ELEMENTARY

Q.1 (4)

A sin θ = x



so θ = π/6

Q.2 (3)

y = a sin (2πnt + α). Its phase at time t = 2πnt + α

Q.3 (2)

Q.4 (1)

Velocity of a particle executing S.H.M. is given by

$$v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{2\pi}{T}\sqrt{\frac{3A^2}{4}} = \frac{\pi A\sqrt{3}}{T}$$

Q.5 (4)

From the given equation, a = 5 and ω = 4

$$\therefore\ v = \omega\sqrt{a^2 - y^2} = 4\sqrt{(5)^2 - (3)^2} = 16$$

$$\therefore\ v = \omega\sqrt{a^2 - y^2} = 4\sqrt{(5)^2 - (3)^2} = 16$$

Q.6 (4)

At mean position velocity is maximum

i.e., $v_{max} = \omega a \Rightarrow \omega = \frac{v_{max}}{a} = \frac{16}{4} = 4$

$\therefore\ v = \omega\sqrt{a^2 - y^2}$ $\Rightarrow 8\sqrt{3} = 4\sqrt{4^2 - y^2}$

$\Rightarrow 192 = 16\,(16 - y^2)$ $\Rightarrow 12 = 16 - y^2$

$\Rightarrow y = 2$ cm

Q.7 (3)

Velocity $v = \omega\sqrt{A^2 - x^2}$ and acceleration = $\omega^2 x$

Now given, $\omega^2 x = \omega\sqrt{A^2 - x^2} \Rightarrow \omega^2.1 = \omega\sqrt{2^2 - 1^2}$

$\Rightarrow \omega = \sqrt{3}\quad \therefore\ T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$

Q.8 (1)

(1) P.E. $= \frac{1}{2}m\omega^2 x^2$

It is clear P.E. will be maximum when x will be maximum i.e., at x = ± A

Q.9 (1)

Since maximum value of $\cos^2\omega t$ is 1.

$\therefore\ K_{max} = K_0 \cos^2 \omega t - K_0$

Also $K_{max} = PE_{max} = K_0$

Q.10 (1)

$$E = \frac{1}{2}ma^2\omega^2 = \frac{1}{2}ma^2\left(\frac{4\pi^2}{T^2}\right) \Rightarrow E \propto \frac{a^2}{T^2}$$

Q.11 (1)

At mean position, the kinetic energy is maximum

Q.12 (2)

Q.13 (4)

Maximum velocity $= a\omega = a\sqrt{\frac{k}{m}}$

Given that $a_1\sqrt{\frac{K_1}{m}} = a_2\sqrt{\frac{K_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{K_2}{K_1}}$

Q.14 (4)

Given spring system has parallel combination, so

$k_{eq} = k_1 + k_2$ and time period $T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

$k_{eq} = k_1 + k_2$ and time period $T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

Q.15 (2)

With respect to the block the springs are connected in parallel combination.

$\therefore$ Combined stiffness $k = k_1 + k_2$ and $n = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$

Q.16 (3)

$$\frac{1}{k_{eff}} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + ...$$

$$= \frac{1}{k}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right] = \frac{1}{k}\left(\frac{1}{1-1/2}\right) = \frac{2}{k}$$

(By using sum of infinite geometrical progression

$a + \frac{a}{r} + \frac{a}{r^2} + ...\infty$ sum (S) = $\frac{a}{1-r}$)

$$\therefore\ k_{eff} = \frac{k}{2}$$

Q.17 (3)

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$$

Q.18 (1)

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \sqrt{\frac{l}{g}} = \text{constant}$$

$$\Rightarrow l \propto g \Rightarrow \frac{l_m}{1}\frac{1}{6}\frac{g}{g} \Rightarrow l_m = \frac{1}{6}\text{m}$$

Q.19 (4)

Frequency $n \propto \frac{1}{\sqrt{l}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}}$

$$\Rightarrow \frac{l_1}{l_2} = \frac{n_2^2}{n_1^1} = \frac{3^2}{2^2} = \frac{9}{4}$$

Q.20 (3)

If first equation is $y_1 = a_1 \sin\omega t$

$$\Rightarrow \sin\omega t = \frac{y_1}{a_1} \qquad ...\text{(i)}$$

then second equation will be $y_2 = a_2 \sin\left(\omega t + \frac{\pi}{2}\right)$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (1)

Comparing $F = -kx$

with $F = -cx^{1/3}$

$\Rightarrow kx = cx^{1/3}$

$\Rightarrow c = kx^{2/3}$

As x increases c also increases.

Q.2 (1)

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

or $a \propto x$

Q.3 (4)

A particle returns back to its original position in one time period.

Q.4 (3)

The total distance moved by particle in one time period is four times the amplitude.

Q.5 (1)

Position where we see the particle once in a time period that is only extreme position. twice through every other position

Q.6 (2)

$\vec{V}_{max}$ only $\vec{V}_{max}$

If initial velocity is $\vec{V}_{max}$

then after one time period particle acquires same speed

V_{max} in same direction means same velocity $\vec{V}_{max}$

Q.7 (2)

A particle appears only once at one of the extreme position in entire oscillation.

Q.8 (3)

In the particle moves from extreme to half of amplitude then let the time taken is t

$y = A\sin(\omega t + \phi)$

$y = A\sin\left(\omega t + \frac{\pi}{2}\right)$ (As particle starts from extreme)

$$\frac{A}{2} = A\sin\left(\frac{2\pi}{4}t + \frac{\pi}{2}\right)$$

$$\sin\left(\frac{\pi}{2}t + \frac{\pi}{2}\right) = \sin\frac{5\pi}{6}$$

$$\frac{\pi}{2}t + \frac{\pi}{2} = \frac{5\pi}{6}$$

$$\frac{\pi}{2}t + \frac{5\pi}{6} = \frac{\pi}{2}$$

$$\frac{\pi}{2}t+\frac{\pi}{3}$$

$$t=\frac{2}{3}\text{sec}$$

Q.9 (3,4)
non-positive means negative scalar product of two vectors will be negative if they are antiparallel. as $\vec{F}$ and $\vec{a}$ are always in same direction and opposite to $\vec{r}$

Q.10 (D)
Vectors in all the pairs are either parallel or anti parallel so angle is either 0º or 180º hence cross product is zero.

Q.11 (2)
$\omega = 3\pi \times 5 = 15\pi, \qquad A = 0.05$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{15\pi} = \frac{1}{7.5}$ sec.Ans.——(B)

$a_{max} = \omega^2 A$
$= (15\pi)^2\ 0.05$
$= 225 \times 0.05$
$= 11.25\ \pi^2$ Ans.——(C)

Q.12 (1)
$y = a \sin(\omega t - kx)$

$y = b\cos(\omega t - kx) \Rightarrow y = b\sin(\omega t - kx + \frac{\pi}{2})$

So phase difference is $\pi/2$

Q.13 (3)
$y = a\cos\omega t$

$\frac{a}{2} = a\cos\omega t$

$\omega t = \frac{\pi}{3}$

$\frac{2\pi}{24}t = \frac{\pi}{3}$

t = 4 sec.

Q.14 (4)
$a = -\omega^2 x$
$= -\omega^2 A\sin\omega t$

$$\langle a\rangle = \frac{-\omega^2 A\int_0^T(\sin\omega t)dt}{\int_0^T dt} = \frac{-\omega^2 A\left[\frac{-\cos\omega t}{\omega}\right]_0^T}{T-0} = 0$$

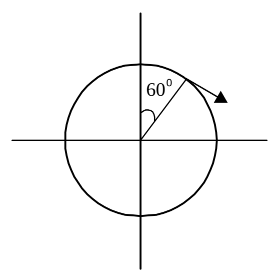
Q.15 (A)

$$a_{avg} = \frac{\int_0^{T/2} -\omega^2 A\sin\omega t\, dt}{\int_0^{T/2} dt} \Rightarrow \frac{-\omega^2 A\left[\frac{\cos\omega t}{\omega}\right]_0^{T/2}}{T/2}$$

$$|a|_{avg} = \frac{-\omega^2 A[-1-1]}{T/2} \Rightarrow \frac{4\omega A}{\frac{2\pi}{\omega}} = \frac{2\omega^2 A}{\pi}$$

Q.16 (1)
Velocity is maximum at mean. To come back to mean the particle has to move $\left(\pi-\frac{\pi}{3}\right)=\frac{2\pi}{3}$. Hence

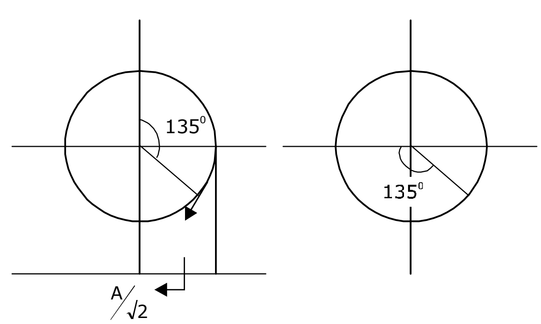
$t = \frac{2\pi}{3.2\pi} = \frac{1}{3}$sec.



Q.17 (1)
$y = 5\sin\pi(t+4) \Rightarrow y = 5\sin(\pi t + 4\pi)$ ---(1)
The standard equation is $y = A\sin(\omega t+\phi)$---(2)
comparing equation (1) & (2)

$A = 5$m, $\omega = \pi$ $\quad T = \frac{2\pi}{\omega} = 2$sec.

Q.18 (3)
Let particle A be the particle shown with initial phase 135° and B be the particle at extreme. Hence the phase difference between them is 135°.



Q.19 (3)

$V = \omega\sqrt{A^2 - x^2}$

$(50\pi)^2 = (10\pi)^2(10^2 - x^2)$

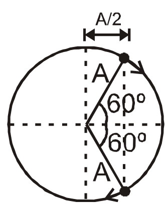
$\Rightarrow \quad x = \pm\sqrt{75} = \pm 5\sqrt{3}$

So, separation between points is

$\therefore \quad \Delta x = 2 \times 5\sqrt{3} = 10\sqrt{3} = 17.32$ cm.

Q.20 (2)

Consider SHM as projection of uniform circular motion. From figure the phase difference between two particles is 120º.



Q.21 (2)

Velocity $v = \omega\sqrt{A^2 - x^2} \Rightarrow v^2 = \omega^2 A^2 - \omega^2 x^2$ ....(1)

Acceleration $a = -\omega^2 x \quad \Rightarrow a^2 = \omega^4 x^2$ ....(2)

From (1) and (2) : $v^2 = \omega^2 A^2 - a^2/\omega^2 \Rightarrow v^2 + a^2/\omega^2 = \omega^2 A^2$

$\Rightarrow \quad \dfrac{v^2}{\omega^2 A^2} + \dfrac{a^2}{\omega^4 A^2} = 1$

$\Rightarrow \quad v^2 = -a^2\left(\dfrac{1}{\omega^2}\right) + 1$ its straight line with -ve slope and +ve intercept

Q.22 (1)

From given equation $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{100\pi} = \dfrac{1}{50} = 0.02$

and A = 0.01

Q.23 (2)

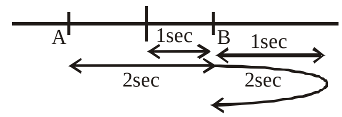
At equilibrium position K.E. is maximum.

Q.24 (3)

$\dfrac{1}{2}kx^2 = \dfrac{1}{2}k(A^2 - x^2)$

or $\quad x = \dfrac{A}{\sqrt{2}}$.

Q.25 (2)



Hence $\dfrac{T}{4} = 2$ sec. T = 8 sec.

Q.26 (2)

$y = 0.45 \sin 2t$

$\dfrac{7.5}{100} = 0.45 \sin 2t$

$\sin 2t = 0.167$

speed = $a\omega\cos 2t$

$= 0.45 \times 2\sqrt{1 - \sin^2 2t}$

$= 0.9 \times 0.98 = 0.87 = 0.15\sqrt{3}$ ms^{-1}

Q.27 (2)

$a_{max} = \omega^2 A$

$(1.57)^2 = \omega^2(1)$

$\omega = 1.57$ rad/sec^2

$\dfrac{2\pi}{T} = 1.57$

T = 4 sec.

Q.28 (2)

From question

$\dfrac{1}{2}m\omega^2 A^2 = 8 \times 10^{-3} \Rightarrow \dfrac{1}{2} \times 0.1 \times \omega^2 \times (0.1)^2 = 8 \times 10^{-3}$

$\Rightarrow \omega = 4$

So, equation of SHM is $x = A\sin(\omega t + \phi) = 0.1 \sin\left(4t + \dfrac{\pi}{4}\right)$.

Q.29 (1)

$x = A\cos\omega t$

K.E. $= \dfrac{1}{2}k(A^2 - x^2)$

$= \dfrac{1}{2}kA^2 \sin^2\omega t$

$= \dfrac{1}{2}kA^2 \dfrac{(1 - \cos 2\omega t)}{2}$

$= \dfrac{kA^2}{4}(1 - \cos 2\omega t)$

Frequency of K.E. is double of acceleration.

Q.30 (1)
Total Energy of S.H.M. remains constant so average energy = Total energy

Q.31 (3)
$X = A\sin 2\pi\upsilon t$,

$$K.E. = \frac{1}{2}K(A^2 - x^2) = \frac{1}{2}K\ (A^2 - A^2\sin^2 2\pi\upsilon t) = \frac{1}{2}K\ A^2$$

$$(\cos^2 2\pi\upsilon t) = \frac{K}{4}A^2(1 + \cos 4\pi\upsilon t)$$

Hence, the frequency of K.E. is 2υ.

Q.32 (3)

$$E_1 = \frac{1}{2}Kx^2$$

$$E_2 = \frac{1}{2}Ky^2$$

$$E = \frac{1}{2}K(x+y)^2$$

$$\Rightarrow\quad E = \frac{1}{2}K(x^2 + y^2 + 2xy)$$

$$= E_1 + E_2 + K\,xy$$

$$= E_1 + E_2 + K\sqrt{\frac{2E_1}{K}}\sqrt{\frac{2E_2}{K}}$$

$$= E_1 + E_2 + 2\sqrt{E_1E_2}$$

Q.33 (1)
$x = A\sin\omega t$ if t = 1 is t=0

$$v = A\omega\cos\omega t \Rightarrow 0.25 = A \times \frac{2\pi}{6}\cos\left(\frac{\pi}{3}(t-1)\right)$$

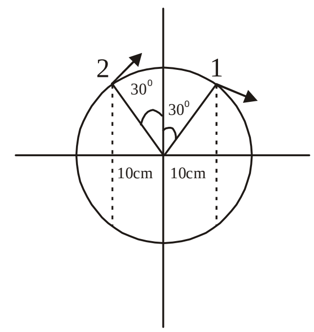
At t = 2 sec.

$$\Rightarrow\ 0.25 = A \times \frac{2\pi}{6} \times \frac{1}{2} \qquad \Rightarrow A = \frac{3}{2\pi}$$

Q.34 (3)

$$\frac{4d^2y}{dt^2} + 9y = 0 \Rightarrow \omega^2 = \frac{9}{4} \Rightarrow \omega = \frac{3}{2}$$

Q.35 (3)



Particle 1 and 2 are as shown and their phase difference is 60°.

Q.36 (2)
Slope of F-x curve gives K

$$\text{slope} = \frac{13.5}{1.5} \quad F = -Kx \Rightarrow K = 9$$

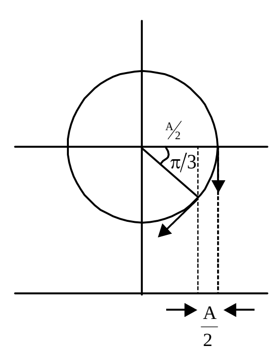
$$\omega^2 = \frac{K}{m} = 9 \quad \omega = 3,\ \ T = 2\pi/3$$

Q.37 (2)
From question

$$\frac{1}{2}m\omega^2A^2 = 8\times10^{-3} \Rightarrow \frac{1}{2}\times0.1\times\omega^2\times(0.1)^2 = 8\times10^{-3} \Rightarrow$$

$\omega = 4$
So, equation of SHM is $x = A\sin(\omega t + \phi) = 0.1\sin\left(4t + \frac{\pi}{4}\right)$.

Q.38 (3)



$$\theta = \pi/3 = \omega t$$

$$t = \frac{T}{6}$$

$$\therefore V_{avg} = \frac{A/2}{T/6} = \frac{3A}{T}$$

Q.39 (1)

$V_{max} = A\omega = V$

ω remains unchanged in this case.

Hence $V_{max_2} = 2A\omega$

$V_{max} = 2V$

Q.40 (4)

$$\frac{1}{2}mv^2 = \frac{1}{2}K_1x_1^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}K_2x_2^2$$

$K_1x_1^2 = K_2x_2^2$

$$\frac{x_1}{x_2} = \sqrt{\frac{K_2}{K_1}}$$

Q.41 (3)

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{K}{m_1}}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{K}{m_2}}$$

$f_2 = \frac{f_1}{2}$ or $m_2 = 4m_1$ or या $m_2 - m_1 = 3$ kg

Q.42 (3)

$T = 2\pi\sqrt{\frac{M}{K}}$ When the rubber ribbon slacks it will not exert any force.

Q.43 (3)

K will remain unchanged

$$\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \text{constant}$$

Q.44 (1)

$\omega = \sqrt{\frac{K}{m}}$ and Ke = mg (at mean position)

or $\omega = \sqrt{\frac{g}{e}}$

Q.45 (4)

$kx = mg \sin 30^o$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g\sin 30^o}{x}} = \sqrt{\frac{5\times 100}{2.5}} = 14.14 \textbf{ Ans.}$$

Q.46 **(4)**

Time period $= T = 2\pi\sqrt{\frac{m}{K}}$

Spring divided into two equal parts so length is reduced to half

We know $K \propto \frac{1}{\ell}$

$\therefore$ K become twice

$$T_{new} = 2\pi\sqrt{\frac{m}{K_{new}}} = 2\pi\sqrt{\frac{m}{2K}}$$

$$= \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{m}{K}}\right) = \frac{T}{\sqrt{2}}$$

Q.47 (2)

$$k_{eq} = 2k + k + \frac{2k\times 2k}{2k+2k} = 4k$$

so, frequency, $f = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M}} = \frac{1}{2\pi}\sqrt{\frac{4K}{M}}$

Q.48 (C)

$$\frac{1}{K_{eq}} = \frac{1}{K} + \frac{1}{2K} + \frac{1}{4K} + \frac{1}{8K} + ...$$

or $K_{eq} = \frac{K}{2}$

Q.49 (D)

$$E = \frac{1}{2}m\omega^2A^2$$

$$= \frac{1}{2}m \times \frac{k}{m} \times A^2$$

$$= \frac{1}{2}kA^2$$

E is independent of mass.

Q.50 (4)

Total M.E.= T.E. at M.P.+Total oscillation energy 9 = 5 + 4

Total oscillation energy = $\frac{1}{2}Ka^2 = 4$

$\Rightarrow K = 8\times10^4 \Rightarrow T = 2\pi\sqrt{M/K} \Rightarrow T = \pi/100$

Q.51 (3)

In spring mass system time period depends only on k and m, not on g

Q.52 (2)

In series $T_1 = 2\pi\sqrt{\frac{2m}{K}}$

In parallel $T_2 = 2\pi\sqrt{\frac{m}{2K}}$

$\frac{T_{series}}{T_{parallel}} = 2$

Q.53 (B)

$V_{max} = \omega A \quad \omega_p A_p = \omega_Q A_Q$

$\frac{A_p}{A_Q} = \frac{\omega_Q}{\omega_p} = \sqrt{\frac{K_2}{K_1}}$

Q.54 (3)

$E_i = \frac{1}{2}KA^2$

$E_i = \frac{1}{2}m\omega^2A^2$

$E_f = \frac{1}{2}(2m)\omega'^2 A^2$

The value of k remains same so $\omega' = \frac{\omega}{\sqrt{2}}$

and hence $E_f = \frac{1}{2}m\omega^2A^2$

So the energy remains unchanged.

Q.55 (3)

$T_p = 2\pi\sqrt{\frac{m}{K}} \quad T_s = 2\pi\sqrt{\frac{\ell}{g+a}}$

T_p = Remain same T_s decreases

Q.56 (1)

We know that

$x = A\sin\omega t$

$\frac{a}{2} = a\sin\omega t$

$\omega t = \frac{\pi}{6}$

$t = \frac{\pi}{6\omega}$

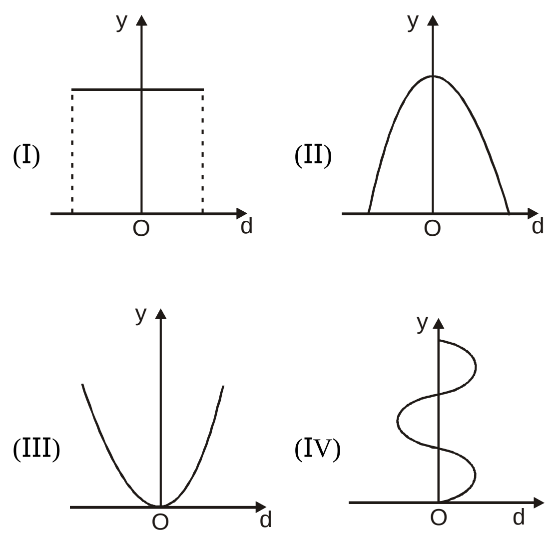
Now $v = a\omega\cos\omega t$

$a\omega\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\frac{a\,2\pi}{T} = \frac{a\pi\sqrt{3}}{T}$

Q.57 (4)

Initially the COM of spere and water lies at centre of sphere .As water flows out the COM shifts down and length of pendulum increases hence time period increases but when water level becomes half of the sphere the COM again starts shifting up and hence as length decreases time period also decreases.

The graph in figure show that a quantity y varies with displacement d in a system undergoing simple harmonic motion.



Which graphs best represents the relationship obtained when y is

Q.58 (4)

$g_{Moon} = \frac{g_{Earth}}{6} \quad \therefore T_{Moon} = \sqrt{6}\ T_{Earth}$

Q.59 (1)

Given time for both are same

$$9T_1 = 7T_2$$

$$9 \times 2\pi\sqrt{\frac{\ell_1}{g}} = 7 \times 2\pi\sqrt{\frac{\ell_2}{g}}$$

$$\Rightarrow 9\sqrt{\ell_1} = 7\sqrt{\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{49}{81}$$

Q.60 (4)

Let $x_1 = A_1 \sin \omega_1 t$ and $x_2 = A_2 \sin \omega_2 t$

Two pendulums will vibrate in same phase again when there phase difference $(\omega_2 - \omega_1)t = 2\pi$

$$\Rightarrow \left(\frac{2\pi}{T_2} - \frac{2\pi}{T_1}\right)t = 2\pi$$

$$\Rightarrow \left(\sqrt{\frac{g}{1}} - \sqrt{\frac{g}{1.44}}\right) n \times T_1 = 2\pi$$ (where n is number of vibrations completed by longer pendulum)

$$\Rightarrow \left(\sqrt{\frac{g}{1}} - \sqrt{\frac{g}{1.44}}\right) n \times 2\pi\sqrt{\frac{1.44}{g}} = 2\pi \Rightarrow n = 5$$

Thus after 5 vibrations of longer pendulum they will again start swinging in same phase.

Q.61 (3)

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{\ell}{g + g/4}} = \frac{2}{\sqrt{5}}T.$$

Q.62 (4)

$T = 2\pi\sqrt{\frac{\ell}{g}}$, As it does not depend on amplitude

$\therefore$ % change in time period is 0 % Hence option (4) is correct.

Q.63 (3)

$$T_{max} = mg + m\omega^2\ell = mg + m\frac{v^2}{\ell} = mg + m\frac{(\omega A)^2}{\ell} = mg + m\frac{g}{\ell} \times \frac{A^2}{\ell} \quad \left(\because \omega = \sqrt{\frac{g}{\ell}}\right)$$

$$\text{or } T_{max} = mg + mg\left(\frac{A}{\ell}\right)^2 = mg\left[1 + \left(\frac{A}{\ell}\right)^2\right]$$

Q.64 (3)

$T = 2\pi\sqrt{\frac{\ell}{g}}$, At high altitude value of g decreases

$\therefore$ length of pendulum must be decreased to keep correct time.

Q.65 (4)

$$I = \frac{2}{5}mR^2 = \frac{2}{5} \times 25 \times (0.2)^2 = \frac{2}{5}$$

$$\tau = C\theta \quad c = \frac{\tau}{\theta} = \frac{0.1}{1} = 0.1$$

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{2}{5 \times 0.1}} = 2\pi \times 2 = 4\pi \text{ secs}$$

Q.66 (1)

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$T_1 = 2\pi\sqrt{\frac{m}{K_1}} \qquad T_2 = 2\pi\sqrt{\frac{m}{K_2}}$$

As kl = constant

Hence $k_1 l_1 = k_2 l_2$

$k_1 L = k_2 2L$

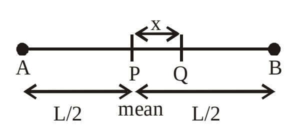
$$\frac{k_1}{k_2} = 2$$

$$\frac{T_1}{T_2} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

Q.67 (2)

Q.68 (1)

Q.69 (2)



Let the particle be displaced a distance x to Q then a quarter of way from A to B means particle is at L/2 from A so time taken is T/6.

$$F_{net} = 2\left(\tfrac{L}{2} - x\right) - \left(\tfrac{L}{2} + x\right) \Rightarrow F_{net} = -4x$$

$$T = 2\pi, \ \omega = 1, \tfrac{T}{6} \Rightarrow \therefore \tfrac{\pi}{3}$$

Q.70 (1)

$n_1T_1 = n_2T_2$ $\qquad n_1T = \frac{5T}{4} \times n_2$

$\frac{n_1}{n_2} = \frac{5}{4} \Rightarrow n_1 = 5, \qquad n_2 = 4$

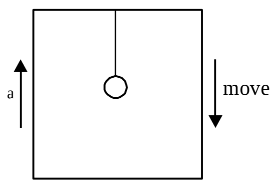
Q.71 (1)

When the lift is going down with constant velocity the acceleration is zero.

When there is a retardation of 'a' the $g_{effective}$ is $g + a$.

V = constant $\Rightarrow a = 0$ So there is no effect.

$T_1 = T = 2\pi\sqrt{\frac{\ell}{g}}$ $\quad g_{eff} = (g + a)$



$T_2 = 2\pi\sqrt{\frac{\ell}{g+a}}$

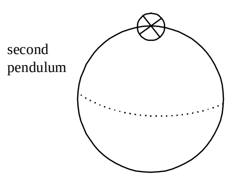
$T_1 > T_2$

Q.72 (1)

T = 2 sec.

$T = 2\pi\sqrt{\frac{I}{mgd}}$

Time period of a second's pendulum is two seconds.



$2 = 2\pi\sqrt{\frac{2mR^2}{mgR}}$ $\quad R = .5m$

Q.73 (3)

$y = A(1 + \cos 2\omega t)$ $\quad y = 2A\sin\left(\omega t + \pi/3\right)$

$y = A + A\cos 2\omega t$ $\quad V_{max} = 2A\omega$

$V_{max} = A \times 2\omega$ Ratio = 1:1

Q.74 (1)

$x = 2\sin\omega t$

$y = 2\sin\left(\omega t + \frac{\pi}{4}\right)$

from Lissajous figures if $\phi = \frac{\pi}{4}$ then the path of particle is an ellipse.

Q.75 (2)

$y = A\sin(\omega t + \phi)$ and $x = A\sin(\omega t + \phi)$ then $y = x$ so path is straight line.

Q.76 (2)

$x = C\sin\omega t + D\sin(\omega t + \pi/2)$

$A_r = \sqrt{C^2 + D^2 + 2CD\cos\frac{\pi}{2}}$ $\quad A_r = \sqrt{C^2 + D^2}$

Q.77 (4)

$y = 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right) = 10\sin(3\pi t + \frac{\pi}{3})$

thus amplitude is 10 m or 1000 cm

Q.78 (2)

$x = A\sin\omega t$, $y = A\cos\omega t$ or $x^2 + y^2 = A^2$

Thus the motion of the particle is on a circle.

Q.79 (2)

$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\,t)$

$y = 2\cos\left(\frac{t}{2}\right)\left[\sin\left(\frac{2001\,t}{2}\right) + \sin\left(\frac{1999\,t}{2}\right)\right]$

$y = \sin(1001\,t) + \cos(1000\,t) + \sin(1000\,t)$
$+ \cos(1999\,t)$

$y = \sin(1001\,t) + \sqrt{2}\sin(1000\,t + \pi/4) + \cos(1999\,t)$

So the given expression is composed by three equation of S.H.M.

Q.80 (3)

$a_1 = 1, a_2 = 1 \quad \theta = \frac{\pi}{3}$

$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\theta} \Rightarrow a = \sqrt{3}$

Q.81 (2)

$E = \frac{1}{2}KA_{eq}^2$ $\quad A_{eq} = \sqrt{A^2 + A^2}$

$E = \frac{1}{2}m\omega^2\left(\sqrt{2}A\right)^2 \quad = \sqrt{2}A$

$E = m\omega^2A^2$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (C)

Restoring force $F = \frac{-du}{dx} = \frac{-d}{dx}(u_0(1-\cos ax))$

$F(x) = -u_0 a \sin ax$

for small angle $\sin ax \approx ax$

$$F = -u_0 a^2 x \Rightarrow \text{acc.} = \frac{-u_0 a^2 x}{m} = -\omega^2 x = \left(\frac{2\pi}{T}\right)^2 \times x$$

So, Time period $T = 2\pi\sqrt{\frac{m}{u_0 a^2}}$

Q.2 (C)

$2A_i\,\omega_i^2 = A_f \omega_f^2$

$A_i \omega_i = A_f \omega_f$

If $\omega_f = 2\omega_i$ and $A_f = \frac{A_i}{2}$

The condition will be satisfied.

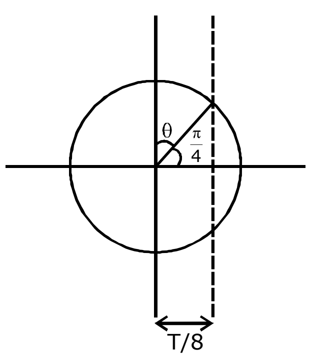
Q.3 (D)

$\theta = \omega t$

$\theta = \frac{2\pi}{T}\cdot\frac{T}{8}$

$\theta = \frac{\pi}{4}$

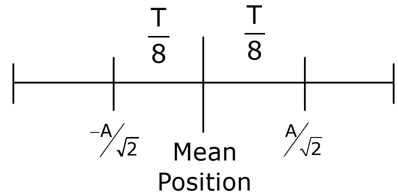
$\cos\frac{\pi}{4} = \frac{x}{A}$



$x = A/\sqrt{2}$

Q.4 (D)

$$\text{Max. Average velocity} = \frac{\text{Total max displacement}}{\text{Total time}}$$



Max^m displacement will be close to $M.P. = \sqrt{2}A$

$$V_{avg} = \frac{4\sqrt{2}}{T}A$$

Q.5 (D)

Time period = 8 sec.

In 1st second $= \frac{2\pi}{8} \times 1$

Displacement $= \frac{A}{\sqrt{2}}$

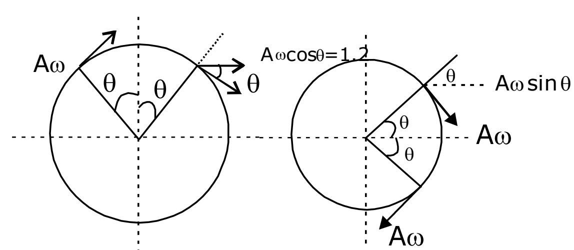
In 1st second $= \frac{2\pi}{8} \times 1$

Displacement $= A = \frac{\pi}{2}$

t = 1

$$\frac{A.\sqrt{2}}{A\sqrt{2}\left(\sqrt{2}-1\right)} = \left(\sqrt{2}+1\right)$$

Q.6 (D)

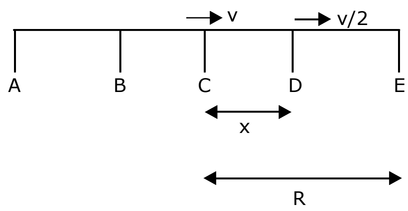


$A\omega\cos\theta = 1.2$ -- (1) $A\omega\sin\theta = 1.6$ -- (2)

$\tan\theta = 4/3 \Rightarrow \theta = 53°$

$\therefore A\omega.\frac{3}{5} = 1.2 \Rightarrow A\omega = 2m/\text{sec}.$

Q.7 (C)



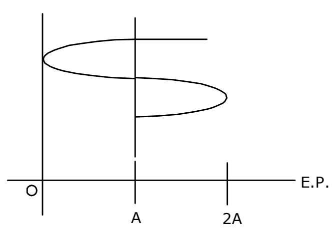
$$v/2 = \omega\sqrt{R^2 - x^2} \quad R\omega/2 = \omega\sqrt{R^2 - x^2}$$

$$R^2/4 = R^2 - x^2 \quad x = \frac{\sqrt{3}}{2}R$$

$$\text{Distance} = 2x = \sqrt{3}R$$

Q.8 (C)

$$x = A + A\sin\omega t$$



$$t = \frac{2.5\pi}{\omega}$$

$$t = \frac{2\pi}{\omega} + \frac{0.5\pi}{\omega}$$

= 5A

Q.9 (C)

A = 2cm. $\Rightarrow x = 1cm = \frac{A}{2}$

$$\Rightarrow \quad v = \omega\sqrt{A^2 - x^2} \quad = \omega\sqrt{3} \quad \Rightarrow \quad a = w^2.1$$

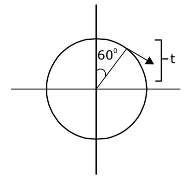
$$\omega = \sqrt{3} = 2\pi n$$

$$\Rightarrow n = \frac{\sqrt{3}}{2\pi}$$

Q.10 (A)

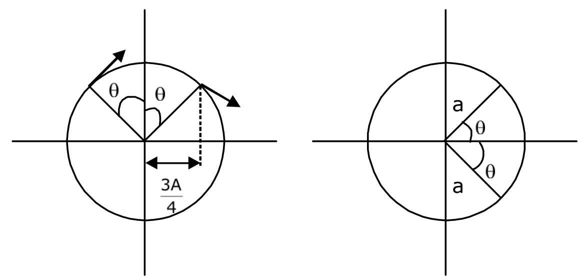
$$y = \sin\omega t + \sqrt{3}\cos\omega t$$

$$y = 2\sin\left(\omega t + \pi/3\right) \qquad A\omega^2 = g$$



$$\Rightarrow \omega = \sqrt{g/2} \Rightarrow t = \frac{\pi}{6}\sqrt{\frac{2}{g}}$$

Q.11 (B)



$$\Rightarrow \cos\theta = \frac{\sqrt{7}}{4}$$

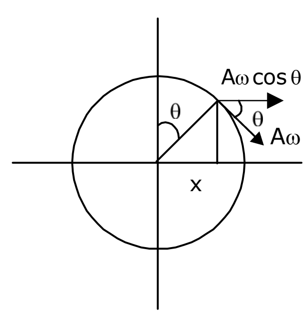
$$\Rightarrow a\cos\theta = \frac{a\sqrt{7}}{4}$$

Q.12 (B)

$$\theta = \omega\frac{T}{12} = \frac{\pi}{6}, \quad x = A\sin\frac{\pi}{6} = \frac{A}{2}$$

$$P.E. = \frac{1}{2}Kx^2 = \frac{1}{2}\frac{KA^2}{4}$$

$$v = A\omega\cos\theta = \frac{A\omega\sqrt{3}}{2}$$



$$KE = \frac{1}{2}mv^2 = \frac{1}{2}K\left(\frac{3}{4}A^2\right) \Rightarrow \frac{KE}{PE} = 3$$

Q.13 (B)

$$y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$$

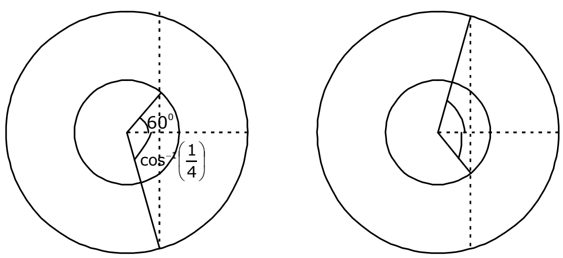
Q.14 (C)

Phase by $\omega = \frac{\omega \times 2\pi}{3\omega} = \frac{2\pi}{3}$

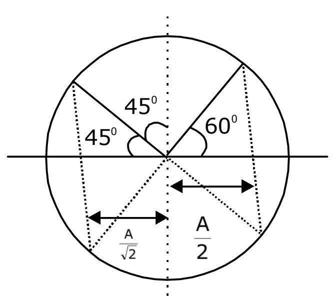
Phase by $2\omega = \phi + 2\omega \times {}^{2\pi}/_{3\omega}$

$, \phi + \frac{4\pi}{3} - \frac{2\pi}{3} = 0, 2\pi, 4\pi \quad \phi = \frac{2\pi}{3}, \frac{4\pi}{3}......$

Q.15 (A)



Q.16 (C)



Q.17 (B)

$v^2 = 108 - 9x^2 \Rightarrow v^2 = 9[12 - x^2]$

$\Rightarrow 2v\frac{dv}{dx} = -2x \times 9 \Rightarrow a = -9x \Rightarrow \omega^2 = 9$

Q.18 (A)

F = -Kx Slope = -K

Q.19 (A)

Acceleration (a) = $\omega^2 x$ $\Rightarrow a_{max} = \left(\frac{2\pi}{T}\right)^2 A$

When $a_{max} = g$

then block and piston will be separated

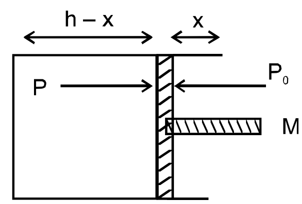
$a_{max} = g = \left(\frac{2\pi}{T}\right)^2 A$; $(g = \pi^2)$

$g = \frac{4\pi^2}{1} A \Rightarrow 33+$

$A = \frac{1}{4} = 0.25\,m$

Q.20 (A)

force $= (P - P_0) A = Ma$



by $P_1V_1 = P_2V_2$

$P_0hA = P(h - x)A \Rightarrow P = \frac{P_0h}{h - x}$

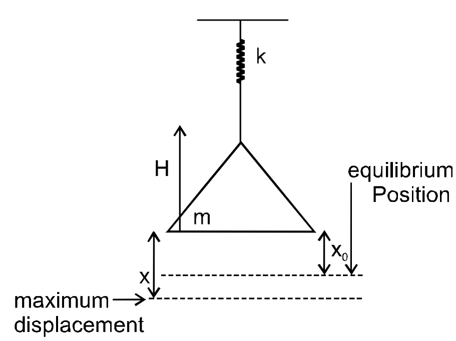
So force $= \left(\frac{P_0h}{h-x} - P_0\right)\frac{A}{M} = a$

$a = \frac{P_0Ax}{M(h-x)} \approx \frac{P_0Ax}{Mh}$; (for small x)

$T = 2\pi\sqrt{\frac{Mh}{P_0A}}$

Q.21 (B)

velocity before collision $= \sqrt{2gH}$



pan is massless so velocity after collision

$= \sqrt{2gH}$

by energy conservation

$$mg(x) + \frac{1}{2} m \left(\sqrt{2gH}\right)^2 = \frac{1}{2} kx^2$$

$$kx^2 - 2mgx - 2mgH = 0$$

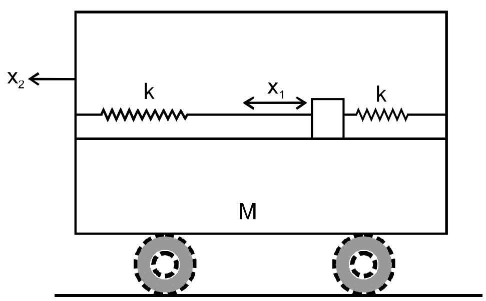
$$x = \frac{mg}{k} + \frac{mg}{k}\sqrt{1+\frac{2Hk}{mg}}$$

at equilibrium $kx_0 = mg \Rightarrow x_0 = mg/k$

mean Amplitude $= x - x_0 = \frac{mg}{k}\sqrt{1+\frac{2Hk}{mg}}$

Q.22 (C)

Let displacement of block is x_1 and of cart is x_2 as shown



by linear momentum conservation

$$mv_1 = Mv_2 \Rightarrow v_2 = \frac{mv_1}{M} \text{ so } x_2 = \frac{mx_1}{M}$$

For block Force equation can be written as

$$F = 2k(x_1 + x_2) = m\omega^2 x_1$$

$$\Rightarrow \quad 2k\left(x_1 + \frac{m}{M}x_1\right) = m\omega^2 x_1 \Rightarrow \omega^2 = 2k\left(\frac{M+m}{Mm}\right)$$

So $T = 2\pi\sqrt{\frac{Mm}{2k(M+m)}}$

Q.23 (B)

$$T_1 = 2\pi\sqrt{\frac{m}{K_1}},\ T_2 = 2\pi\sqrt{\frac{m}{K_2}}\quad T = 2\pi\sqrt{\frac{m}{K_{eq}}}$$

$$K_{eq}\frac{K_1K_2}{K_1+K_2} \text{ (for series) } t_1^2 = \frac{2\pi m}{K_1},\ t_2^2 = \frac{2\pi m}{K_2}$$

and $T^2 = 2\pi m\left(\frac{1}{K_1}+\frac{1}{K_2}\right)$

$$\Rightarrow T^2 = t_1^2 + t_2^2$$

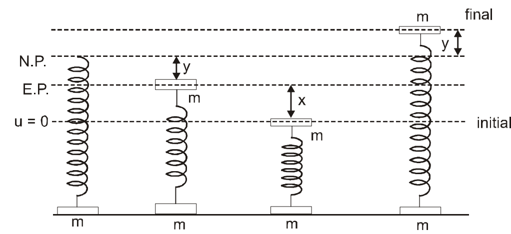
Q.24 (A)

Time period $T = 2\pi\sqrt{\frac{\mu}{k}}$ where $\mu = \frac{M_1M_2}{M_1+M_2}$

$$= \frac{M\times M}{M+M} = \frac{M}{2}$$

So, time period $T = 2\pi\sqrt{\frac{M}{2\alpha}}$ $(\because k = \alpha)$

Q.25 (C)



Let upper block is pushed down by x. at equilibrium mg = ky, i.e., weight of upper block is balanced by spring when it is deformed by y. upper block will perform SHM with amplitude x about equilibrium position. lower block will leave surface when spring is extended by y, means upper block is at distance 2y from its mean position. That should be upper extreme position of upper block. So amplitude x = 2y

Alternate

at equilibrium of upper block mg = ky

lower plate will leave the surface if the extension in spring is y

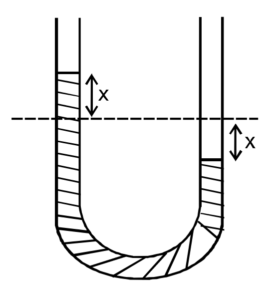
Let upper plate is displaced by x downward and left so by energy conservation between compressed to extended positions

$$0 + \frac{1}{2} k (x + y)^2 = mg (x + 2y) = \frac{1}{2} ky^2$$

$$\Rightarrow \frac{1}{2} kx^2 + \frac{1}{2} ky^2 + kxy = mgx + mg2y + \frac{1}{2} ky^2$$

$$\Rightarrow x = 2y$$

Q.26 (D)



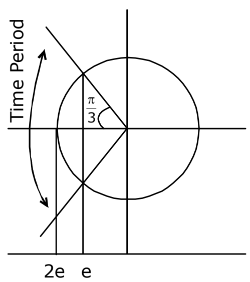
liquid is displaced by x
at displaced position
So net force on fluid = (2xA)ρg = $m\omega^2 x$
$= \rho A (2\ell)\omega^2 x$

$$\omega = \sqrt{\frac{g}{\ell}} \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g}}$$

Q.27 (A)

$$\omega t = \frac{2\pi}{3} \Rightarrow \frac{2\pi}{T}t = \frac{2\pi}{3}$$

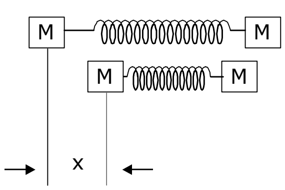
$$\Rightarrow t = \frac{T}{3} \Rightarrow t = \frac{2\pi}{3}\sqrt{\frac{m}{K}}$$



Q.28 (C)

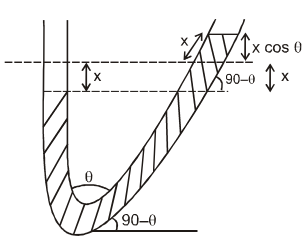
$$\mu = \frac{m_2}{2m} = \frac{m}{2}$$

It can be considered as a two block system for $\frac{T}{2}$ time



$$t = \frac{T}{2} = \pi\sqrt{\frac{m}{2K}} \quad \Rightarrow t = \pi\sqrt{\frac{2}{2.\pi^2}} = 1s.$$

Q.29 (C)

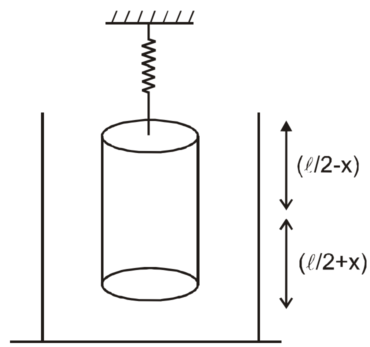


Force on fluid
$F = -\rho sg\,(x + x\cos\theta)$
$= -m\omega^2 x$
$\Rightarrow \quad m\omega^2 = \rho sg\,(1 + \cos\theta)$

$$\Rightarrow \quad \omega = \sqrt{\frac{\rho sg(1+\cos\theta)}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{\rho s(1+\cos\theta)g}}$$

Q.30 (D)



If cylinder is displaced x vertically down as shown then Net force on cylinder
F = kx + x.a.ρg

$$m\frac{d^2x}{dt^2} = (k + a\rho g)\,x \qquad \Rightarrow \frac{d^2x}{dt^2} = \frac{(K + a\rho g)x}{m}$$

$= \omega^2 x = (2\pi n)^2\ x$

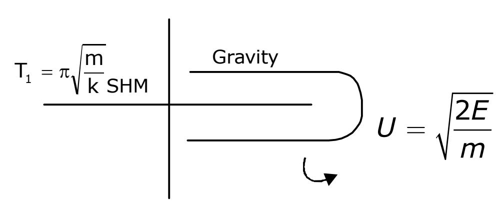
$$\text{So frequency } n = \frac{1}{2\pi}\sqrt{\left(\frac{K + a\rho g}{m}\right)}$$

Q.31 (C)

$$K_{A_{eq}} = K/3,\ K_B = 3K \Rightarrow \frac{T_A}{T_B} = \sqrt{\frac{K_B}{K_A}} = \frac{3}{1}$$

Q.32 (C)
for x < 0 perform SHM

$$\frac{1}{2}mU^2 = E$$

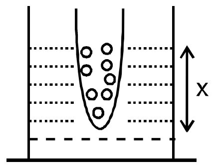


$$\Rightarrow T_2 = \frac{2}{g}\sqrt{\frac{2E}{m}}$$

$$\therefore T = T_1 + T_2 \Rightarrow T = \pi\sqrt{\frac{m}{K}} + \frac{2}{g}\sqrt{\frac{2E}{m}}$$

Q.33 (B)
at equilibrium position
$Ax\rho g = Mg$ $\qquad M = Ax\rho$ ——(1)
If tube displaced by distance y vertically downward

then net force on tube



$A(x+y)\rho g - Mg = Ma$

$a = \frac{A\rho g}{M}\, y = \frac{Mg}{xM}\, y \quad \Rightarrow a = (g/x)y$

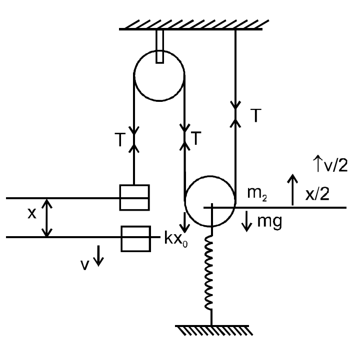
Time period $T = 2\pi\sqrt{x/g}$

Q.34 (A)

$$2 \times \frac{1}{2}KA^2 = \frac{1}{2}KA^2 \Rightarrow A' = \sqrt{2}A$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{K}}$$

Q.35 (C)
(i) Equilibrium position determination
$M_1g = T \rightarrow$ (from FBD of M_1)
$2T = M_2g + kx_0$ $\qquad$ (from FBD of M_2)



$\therefore$ $2M_1g = M_2g + kx_0$
$\therefore$ $kx_0 = 2M_1g - M_2g$
(ii) Displace block M_1 by small disp. x by
At new displaced position

$$-Mgx + \frac{1}{2}M_1V^2 + \frac{1}{2}M_2\left(\frac{v}{2}\right)^2 + M_2g\left(\frac{x}{2}\right) + \frac{1}{2}K\left(x_0 + \frac{x}{2}\right)^2 = C$$

Differentiating equation

$$-M_1g\frac{dx}{dt} + \frac{1}{2}M_1\,2v\frac{dv}{dt} + \frac{M_2}{8}\,2v\frac{dv}{dt} + \frac{M_2g}{2}\frac{dx}{dt} + \frac{K}{2}\,2\left(x_0 + \frac{x}{2}\right)\left(\frac{1}{2}\frac{dx}{dt}\right) = 0$$

$$\Rightarrow -M_1g + M_1a + \frac{M_2a}{4} + \frac{M_2g}{2} + \frac{K}{2}\left(x_0 + \frac{x}{2}\right) = 0$$

(where $a = \frac{dv}{dt}$)

$$\Rightarrow -M_1g + \frac{M_2g}{2} + M_1a + \frac{M_2a}{4} + \frac{Kx_0}{2} + \frac{Kx}{4} = 0 \text{(from}$$

equilibrium $-M_1g + \frac{M_2g}{2} + \frac{Kx_0}{2} = 0$)

Hence, $\frac{4M_1 + M_2}{4}\, a = \frac{-Kx}{4}$

$\therefore$ $a = -\left(\frac{K}{4M_1 + M_2}\right)x \qquad \omega^2 = \frac{K}{(4M_1 + M_2)}$

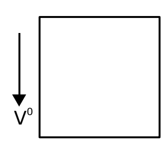
$\omega = \sqrt{\frac{K}{(4M_1 + M_2)}}$; $T = \frac{2\pi}{\omega}$

$\therefore$ $T_2 = 2\pi\sqrt{\frac{4M_1 + M_2}{K}}$

Q.36 (B)

$T = 2\pi\sqrt{\frac{m}{K}}$ ----> not dependent on q_{eff}.

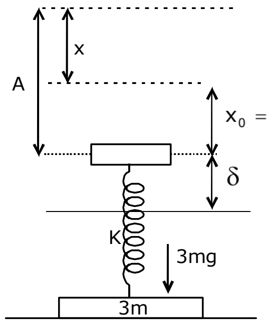
the velocity of particle at M.P. = V_0 $\therefore V_0 = A\omega_0$



$$A = \frac{v_0}{\omega_0}$$

Initial phase is zero.

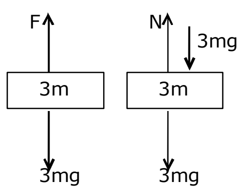
Q.37 (A)



$kx \geq 3mg$ $\delta = \dfrac{2mg}{k}$

$x \geq \dfrac{3gm}{k}$ $F = k\delta + kx = 3mg$

$A = \dfrac{4mg}{k}$



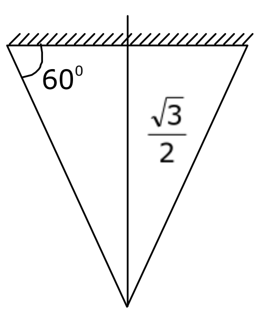
N = 6mg

F $\geq$ 3mg

Q.38 (B)

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

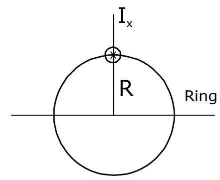
$$= 2\pi\sqrt{\frac{m\left(\frac{\sqrt{3}}{2}\ell\right)^2}{mg\frac{\sqrt{3}}{2}\ell}}$$



$$= 2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$$

Q.39 (C)

$$I_x = \frac{mR^2}{2} + mR^2$$



$$= \frac{3}{2}mR^2$$

$$T = 2\pi\sqrt{\frac{3}{2}\frac{mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} \Rightarrow \ell = \frac{3}{2}$$

Q.40 (A)

For simple pendulum $\omega = \sqrt{g/\ell}$ and maximum linear displacement $x_0 = \ell\theta$ and equation of S.H.M

$x = x_0 \cos \omega t$

$x = \ell\theta \cos \sqrt{g/\ell}\, t$

Q.41 (A)

$$T_1 = 2\pi\sqrt{\ell/g} = T \Rightarrow T_2 = 2\pi\sqrt{\ell/4g} = \frac{T}{2}$$

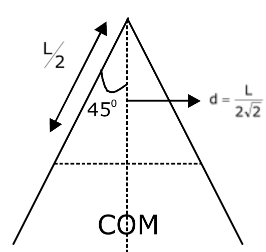
$$T_{eq} = \frac{T_1}{2} + \frac{T_2}{2} \Rightarrow \quad = \frac{T}{2} + \frac{T}{4} = \frac{3T}{4}$$

Q.42 (B)

$$T = 2\pi\sqrt{\frac{I}{mgd}} \quad I = \frac{2md^2}{3}$$

$$T = 2\pi\sqrt{\frac{2md^2}{3}\frac{2\sqrt{2}}{(2m)gd}}$$

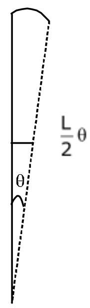
$$T = 2\pi\sqrt{\frac{2\sqrt{2}d}{3g}}$$



Q.43 (C)

$$KL^2\theta + \frac{KL^2\theta}{4} > mg.\frac{L}{2}.\theta$$

K> 2mg/5L



Q.44 (A)

$$\tau_{net} = KL^2\theta + \frac{KL^2\theta}{4} - mg.\frac{L\theta}{2} \quad \because K = mg/L$$

$$I = mL^2/3 \Rightarrow \tau_{net} = \frac{3}{4}KL^2\theta \Rightarrow K_{eq} = \frac{3}{4}KL^2$$

$$\omega = \sqrt{\frac{K_{eq}}{I}} = \frac{3}{2}\left(\frac{K}{m}\right)^{1/2}$$

Q.45 (D)

$$T_A = 2\pi\sqrt{\frac{I_A}{mgx}} \quad \text{----(1)}$$

$$T_B = 2\pi\sqrt{\frac{I_B}{mg(0.25-x)}} \quad \text{-----(2)}$$

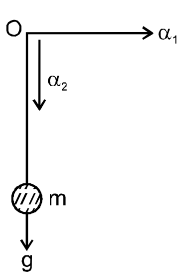
Eq. (1) & (2)

$$\frac{3}{4} = \sqrt{\frac{I_A(0.25-x)}{I_B x}} \quad \Rightarrow \frac{3}{4} = \sqrt{\frac{9}{4}\left(\frac{0.25-x}{x}\right)}$$

$$\Rightarrow \frac{9}{16} = \frac{9}{4}\left(\frac{0.25-x}{x}\right) \Rightarrow x = 0.20\text{m}$$

Q.46 (A)

Point O is moving as shown



So acc. Of particle w.r.t O

$$= (-\alpha\hat{i} + (\alpha_2 - g)\hat{j}$$

So $g_{eff.} = \sqrt{\alpha_1^2 + (g-\alpha_2)^2}$

So time period

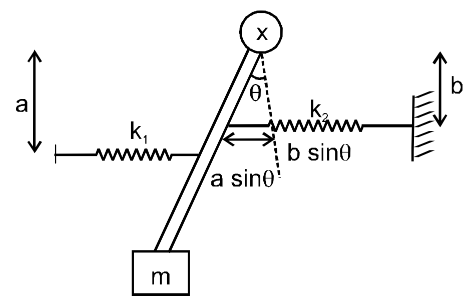
$$= 2\pi\sqrt{\frac{\ell}{(\alpha_1^2 + (g-\alpha_2)^2)^{1/2}}}$$

Q.47 (A)

For small angular displacement (θ)

Net torque on body = Iα

$$= (k_1 a \sin\theta)a + (k_2 b \sin\theta)b = \left(mL^2 + \frac{ML^2}{3}\right)\alpha$$



For small $\theta \Rightarrow a = \dfrac{k_1a^2 + k_2b^2}{mL^2 + \dfrac{ML^2}{3}}\theta$

$$\Rightarrow \text{frequency} = \frac{1}{2\pi}\sqrt{\frac{k_1a^2 + k_2b^2}{L^2(m + M/3)}}$$

Q.48 (B)

$$x = 10\sin^3(\pi t)\ ; \quad \left(\sin^3 A = \frac{3\sin A - \sin(3A)}{4}\right)$$

$$\Rightarrow x = 10\left[\frac{3\sin(\pi t) - \sin(3\pi t)}{4}\right]$$

$$\Rightarrow x = \frac{30}{4}\sin\pi t - \frac{10}{4}\sin 3\pi t$$

So Amplitude = $\frac{30}{4}$, $\frac{10}{4}$

frequency = 1/2 , 3/2

JEE-ADVANCED
MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A, D)
$x = x_0 \sin^2 wt = \dfrac{x_0(1-\cos 2\omega t)}{2}$

Q.2 (B,C,D)

$$mg = Kx_o \quad x_o = \frac{10}{500} = 2cm$$

$$w = \sqrt{\frac{500}{1}} = 10\sqrt{5}\ \frac{rad}{sec}$$

$\therefore$ Maximum velocity = Aw = $3\times10^{-2} \times 10\sqrt{5}$

$= 30\sqrt{5}$ cm/sec. $\Rightarrow$ Max^m Acc^n = Aw^2 = 15 m/s^2

Q.3 (A, C)

$V = V_0 \cos wt = V_0 \cos \dfrac{2\pi t}{T}$

at $\tau = \dfrac{T}{6}$

$\varsigma = V_0 \cos \dfrac{2\pi}{T} . \dfrac{T}{6} = \dfrac{V_0}{2}$ (A)

$a = a_0 \sin wt = a_0 \sin \left(\dfrac{2\pi t}{T}\right)$

$t = \dfrac{T}{6}$

$a = a_0 \sin \left(\dfrac{2\pi}{T} T/6\right) = \dfrac{\sqrt{3}a_0}{2} = 0.86\ a_0$

(C)

Q.4 (B,C,D)

$u = 5x^2 - 20x \Rightarrow F = \dfrac{-du}{dx} = -10(x-2)$

M.P. aτ x = 2m

Q.5 (B,C)

KE Average = $\dfrac{1}{4}KA^2 = m\pi^2 f^2 A^2$

PE Average = $m\pi^2 f^2 A^2$

Q.6 (A,B,C,D)
$Aw^2 = g \Rightarrow 0.40\ w^2 = 10$
b) Negative Extreme

Q.7 (A, B, D)
In S.H.M direction of v and a may be same or oppositε and also direction of y and v maψ be same but direction of a and y never same because $a = -w^2y$ (always opposite to y).

Q.8 (B,C)

$$\frac{1}{2}KA^2 = \frac{1}{2} \times 2 \times 10^6 \times 10^{-4} = 100$$

$\therefore 60\ J$ P.E. at M.P.

$$Max^m K.E. = 100\ joule$$

$$Max^m P.E. = 160\ joule$$

Q.9 (A,B,C)

$$x = 5\sin\left(4\pi t + \tan^{-1} 4/3\right)$$

Q.10 (A, B, C)
Equation of S.H.M $x - x_0 = a \sin(wt + f) \Rightarrow x = x_0 + a \sin(wt + f)$
(A) x = sin2wt

(B) $x = \sin^2 wt = \left(\dfrac{1-\cos 2\omega t}{2}\right) = \dfrac{1}{2} - \dfrac{1}{2}\cos 2\ wt$

$\Rightarrow x - \dfrac{1}{2} = -\dfrac{1}{2}\cos 2\ wt$

(C) x = sin wt + 2cos wt = sin (wt + f) $\{f = \tan^{-1} 2\}$
So represent equation of S.H.M

(D) But σin ωt + cos2 wt cannot write as sin (wt+f)

sin wt + cos2 wt dks sin (wt+f)
So it is not S.H.M equation

Q.11 (B,D)

$$x = 3\sin 100t + 4(1 + \cos 100t)$$

$$x = 5\sin\left(100t + \tan^{-1} 4/3\right) + 4$$

M.P. is at 4 with A = 5

Q.12 (A,C,D)

$\vec{r} = A\cos wt\ \hat{i} + 2A\cos wt\ \hat{j} \Rightarrow x = A\cos wt$, y = 2A cos wt so y = 2x

$|\vec{r}|$ = Acoσ wt . $\sqrt{5} = \sqrt{5}$ A cos wt

So motion is at straight livε, perioδic and S.H.M

Q.13 (B,C,D)

$v^2 = \omega^2 (A^2 - x^2)$ F=-Kx

$$\frac{v^2}{\omega^2} + x^2 = A^2 \quad \text{---- Ellipse}$$

$v^2 = \left(A^2 - \omega^2 x^2\right)$, a=-w^{2}x, a^2= w^{4}x^2

$$v^2 = \left(A^2 - \frac{a^2}{\omega^2}\right) \text{ Þ } v^2 + \frac{a^2}{\omega^2} = A^2 \quad \text{---- Ellipse}$$

Q.14 (A,B,C)

v = wA & $\omega = \frac{10}{2.5} = 4$

(a) $T = \frac{2\pi}{\omega} = \frac{\pi}{2} = 1.57$

(b) $a = \omega^2 A = 40$

(c) $v = \omega\sqrt{A^2 - x^2} = 2\sqrt{21}$

Q.15 (B, D)

accelepation a = w^{2}x

maξ^m acceleration $a_{\mu ax} = w^2 A = \left(\frac{2\pi}{T}\right)^2 \frac{2.5}{100}$

When block and platform are separated

$a_{max} = g = 10$

$\frac{4\pi^2}{T^2} \cdot \frac{1}{40} = 10 \quad \Rightarrow T^2 = \frac{\pi^2}{100}$ T = p/10sec

Q.16 (B,C)

Given y = (sin ωt + cos ωt)y = $\sqrt{2}$ sin (ωt + π/4)

So A = $\sqrt{2}$ m and at t = 0 y = 1m

Q.17 (A,C)

$$v = \sqrt{\frac{900}{3}}\sqrt{\left(2^2 - 1^2\right)} = 10\sqrt{3} \times \sqrt{3} = 30 \text{ m/s}$$

M.C. 3 x 30= 9 x v Þ v= 10 m/s

$$10 = \sqrt{\frac{900}{9}}\sqrt{\left(A^2 - 1^2\right)} \Rightarrow A = \sqrt{2}m$$

Q.18 (A)

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{200} \text{ rad/s}$$

Q.19 (C)

Q.20 (D)

(18 to 20)

Maximum extension the spring from natural position is x.

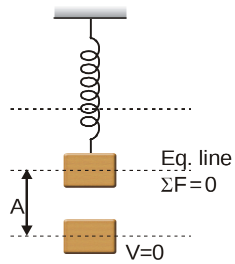
Then mg + ma = kx

$$\Rightarrow x = \frac{2(10+5)}{400} = 7.5 \text{ cm}$$

Extension of the spring when it is stretched to equilibrium line is x'.

mg =kx'

$$\Rightarrow x' = \frac{2 \times 10}{400} = 5 \text{ cm}$$



Therefore amplitude A = x – x' = 2.5 cm

If upward direction is taken as positive at t = 0, x = – A

Using x = A sin (ωt + φ)

– A = A sin φ

$$\phi = \frac{3\pi}{2}$$

Q.21 (B, C, D)

Option 'A' is wrong because it is not given $\omega_1 = \omega_2$

In option 'B' the resultant displacement is given as x =

$$\left[2A_1 \cos\frac{(\omega_1 - \omega_2)}{2}t\right] \cos\left(\frac{\omega_1 + \omega_2}{2}\right)t$$

In option 'C' the resultant amplitude is zero.

In option 'D' the resultant amplitude is $A_1 + A_2 = 2A$.

Q.22 (A, B, C)

In option 'A' the resultant displacement 'r is given as

$r = [A_1^2 + A_3^2]^{1/2} \cos\omega_1 t$ along line $y = \frac{A_3}{A_1}x$

$r = [A_1^2 + A_3^2]^{1/2} \cos\omega_1 t = \frac{A_3}{A_1}x$

In option 'B' $x = A_1 \cos\omega_1 t$

$y = A_1 \cos\left(\omega_1 t + \frac{3\pi}{2}\right)$ or $y = A_1 \sin\omega_1 t$

Thus, $x^2 + y^2 = A_1^2$

$$V_x = \frac{dx}{dt} = -A_1\omega_1 \sin\omega_1 t$$

$$V_y = \frac{dy}{dt} = A_1\omega_1 \cos\omega_1 t$$

at t = 0, $V_x = 0$, V_y = +ve hence the motion is in counter clockwise direction

In option 'C' $\frac{x^2}{A_1^2}+\frac{y^2}{A_3^2}=1$

$x = A_1 \cos\omega_1 t$ $y = A_3 \cos(\omega_1 t + \pi/2) = -A_3 \sin\omega_1 t$
$V_x = -A_1\omega_1 \sin\omega_1 t$ and $V_y = -A_3\omega_1 \cos\omega_1 t$
velocity at t = 0 $V_x = 0$, $V_y = -ve$ at $x = A_1$, y = 0 so motion is clockwise
In option 'D' the motion is SHM along a straight line.

Q.23 (B)
Let $\omega_1 = \omega_2 = \omega$
The resultant SHM is given as
$x = A\sin(\omega t + \alpha)$

where $\tan\alpha = \frac{A_2\sin\delta_2}{A_1 + A_2\cos\delta_2}$.

Q.24 (B)

$K_{eq} = k_1 + k_2 + \frac{k_3k_4}{k_3+k_4}$

$= 20 + 30 + \frac{60\times30}{60+30} = 70$ N/m

Q.25 (B)
$k_{eq} = 70$ N/m
m = 0.7 kg

$\omega = \sqrt{\frac{k_{eq}}{m}} = 10$

$v_{max} = A\omega$
$1 = A\times10$
A = 0.1 m

Q.26 (A)

$T = \frac{2\pi}{\omega} \Rightarrow \frac{2\pi}{10}$

at x = 0 v is positive and y = 0

$\therefore$ (A) is correct

Q.27 (B)
$m_1v_1 = m_2v_2$
$0.7\times1 = 2.8\times v_2$
$v_2 = 0.25$ m/s
$A'\omega' = 0.25$

$\omega' = \sqrt{\frac{k_{eq}}{m_2}}$

= 5

$\therefore$ A' = 0.05 m]

Q.28 (C)

Q.29 (B)

Q.30 (B)
Equilibrium position has shifted by $x_0 = A$
also = $kx_0 = ma$
$70\times0.1 = 0.7\times a$
$a = 10\ m/s^2$

disatance moved by train in 10 sec = $\frac{1}{2}at^2$

$= \frac{1}{2}\times10\times(10)^2$

= 500 m

Q.31 (B)

Q.32 (A)

$E = \frac{1}{2}m\times A^2\omega^2$

$= \frac{1}{2}\times0.7\times0.01\times100$

50% of energy is consumed in melting

$\therefore 0.35\times\frac{50}{100} = 3.36\times10^5$

$\therefore m \approx 5\times10^{-7}$ kg

Q.33 (A) r , (B) p , (C) q , (D) s
$V_{max} = A\omega$

$\Rightarrow A = \frac{V_{max}}{\omega} = \frac{2\pi}{2\pi}$ x (0.2) = 0.20m

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2k}{4\pi^2} = 0.2$ kg

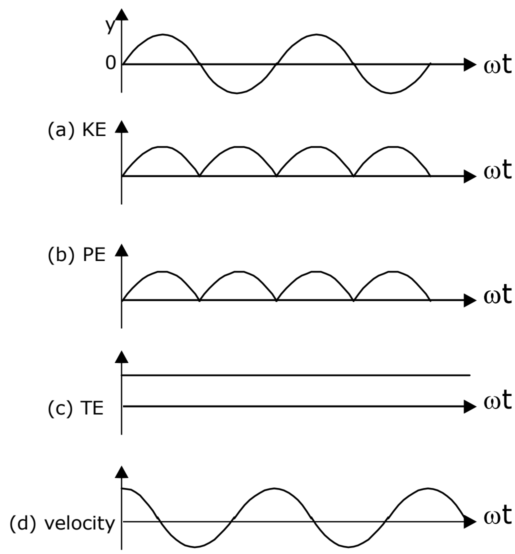
At t = 0.1, acc. is maximum

$\Rightarrow a_{max} = -\omega^2A = -\left(\frac{2\pi}{0.2}\right)^2\times0.2$

$= -200\ m/s^2$

Maximum energy = $\frac{1}{2}mV_{max}^2 = 4$ J

Q.34 (B)



(C) $T = 2\pi\sqrt{\dfrac{I}{mgd}} = 2\pi\sqrt{\dfrac{\frac{m\ell^2}{3}}{mg\frac{\ell}{2}}} = 2\pi\sqrt{\dfrac{2\ell}{3g}}$

(D) $T = 2\pi\sqrt{\dfrac{m}{\rho Ag}} = 2\pi\sqrt{\dfrac{(\rho/2)A\ell}{\rho Ag}} = 2\pi\sqrt{\dfrac{\ell}{2g}}$

Q.35 (A) q,r (B) q,r (C) p (D) s

The velocity-displacement relation is $x^2 + \dfrac{v^2}{\omega^2} = A^2$

$$\Rightarrow \frac{x^2}{A^2} + \frac{v^2}{\omega^2 A^2} = 1$$

which may be a circle if $\omega = 1$. and ellipse if $\omega \neq 1$.
The acceleration – velocity relation is

$\dfrac{v^2}{\omega^2 A^2} + \dfrac{a^2}{\omega^4 A^2} = 1$, which maybe a circle if $\omega = 1$ and ellipse of $\omega \neq 1$.

Acceleration-displacement graph is straight and acceleration time graph is sinusoidal.
(since $a = -\omega^2 x = -\omega^2 A \sin(\omega t + \phi)$)

Q.36 (A) p (B) q (C) p (D) s

(A) In frame of lift effective acceleration due to gravity is $g + \dfrac{g}{2} = \dfrac{3g}{2}$ downwards

$$\therefore T = 2\pi\sqrt{\frac{2\ell}{3g}}$$

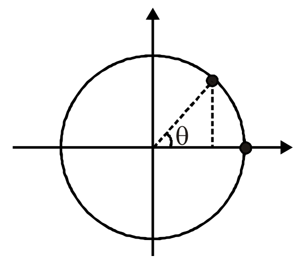
(B) $K\ell = mg \therefore \dfrac{k}{m} = \dfrac{g}{L}$

constant acceleration of lift has no effect in time period of oscillation.

$$\therefore T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\ell}{g}}$$

NUMERICAL VALUE BASED

Q.1 [x = 2]



$$A\cos\theta = \frac{A}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore t = \frac{\theta}{2\pi} T = \frac{1}{6}(4) = \frac{2}{3} \text{ sec.}$$

Q.2 [4]

The bob will execute SHM about a stationary axis passing through AB. If its effective length is 'ℓ' then T

$$= 2\pi\sqrt{\frac{\ell'}{g'}}$$

$$\ell' = \frac{\ell}{\sin\theta} = \sqrt{2}\ell$$

$$g' = g\cos\theta = g/\sqrt{2}$$

$$T = 2\pi\sqrt{\frac{2\ell}{g}} = \frac{2\pi}{5} = \frac{4\pi}{10}$$

x = 4.

Q.3 [0005]

Let the piston are displaced by 'x'
Process isothermal

$$\therefore \Delta P = \frac{P}{V}\Delta V$$

$$\therefore a = \frac{\Delta P(A_1 - A_2)}{m}$$

[A_1 : Area of upper piston
m = 1.5 kg]

$$= \frac{P}{mV}(A_1 - A_2).\Delta V$$

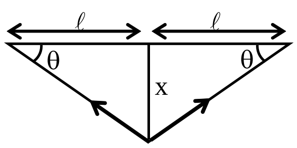
$$= \frac{P}{mV}(A_1 - A_2)^2 x$$

$$\Rightarrow \omega = \sqrt{\frac{P}{mV}}\,.\,(A_1 - A_2)$$

$\Rightarrow$ T = **0.5 sec**

Q.4 [6]

$$F = -4F_0 \sin\theta = -\frac{4F_0 x}{\ell}$$



Q.5 [1]

For stable equilibrium $h \le r \Rightarrow \le 1$

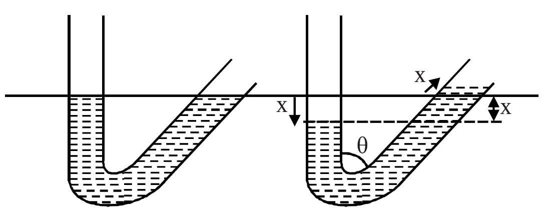
Q.6 [7]

$$P_0 + \rho_2 gh + \rho_1 g(h-20)] = P_0 + \rho_1 V^2$$

$$\Rightarrow V = \sqrt{\frac{\rho_2 gh + \rho_1 g(h\text{-}20)]}{\rho_1}}$$

$$= 4 \text{ m/sec}$$

Q.7 [8 sec]



$$m\omega^2 x = \rho A\left[x + \frac{x}{\cos\theta}\right] g\cos\theta$$

$$m\omega^2 x = \text{rg Ax } (1+\cos\theta)$$

$$\omega = \sqrt{\frac{\rho Ag(1+\cos\theta)}{m}}$$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{\rho Ag(1+\cos\theta)}}$$

$$= 2\sqrt{\frac{m}{\rho A(1+\cos\theta)}} \qquad [\because p^2 \approx g]$$

$$= 2\sqrt{\frac{200\times^{-3}}{13.6\times10^3\times.5\times10^{-4}\left(1+\frac{\sqrt{3}}{2}\right)}}$$

= 0.8 sec

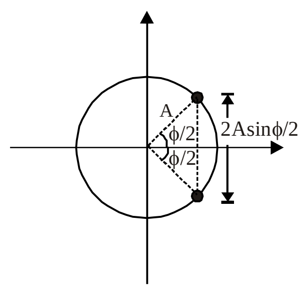
10 T = 8 sec

Q.8 [3]

$$T = 2\pi\sqrt{\frac{M}{K}},\ K \propto 1/l \qquad \therefore K_1 = 4K \ \&\ K_2 = 4/K$$

$$T_1 = 2\pi\sqrt{\frac{M}{4K}} \qquad T_2 = \sqrt{\frac{M}{(4K/3)}}$$

Q.9 [18]



$$\text{Maximum distance} = 2A\sin\frac{\phi}{2} = (2A)\,(0.9) = 1.8\text{ A}$$

Q.10 [60N]

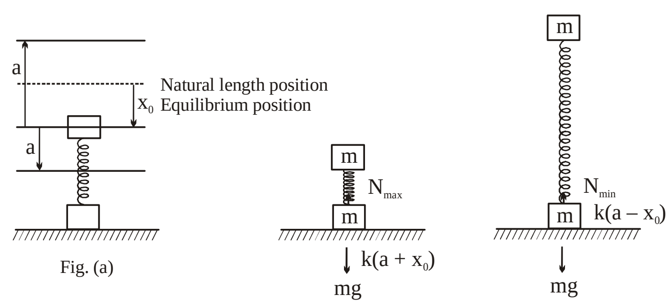


Fig. (a) Fig. (b) Fig. (c)

For equilibrium

$1\times g = kx_0$ where k is spring constant.

Given $\omega = 25$ rad/sec.

Here $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{1}} \quad \Rightarrow \ k = \omega^2 = 625$ rad²/sec²

Fig. (b)

For the maximum force spring should be maximum compressed.

$\therefore \quad N_{max} = 4g + k\,(x_0 + a)$

$= 40 + kx_0 + ka$

$= 40 + 10 + 625 \times \dfrac{1.6}{100}$

$= 60$ newton

KVPY

PREVIOUS YEAR'S

Q.1 (A)

In SHM

$a = -\omega^2 x$

$$v = \omega\sqrt{A^2 - x^2}$$

$$v^2 = \omega^2\left(A^2 - x^2\right)$$

$$v^2 = \omega^2 A^2 - \omega^2 \times \frac{a^2}{\omega^2}$$

$$v^2 + \frac{a^2}{\omega^2} = \omega^2 A^2$$

$$\frac{v^2}{\omega^2 A^2} + \frac{a^2}{\omega^4 A^2} = 1$$

i.e. ellipse

Q.2 (A)

Q.3 (D)

$mg\sin\theta - F_r = ma$

$$F_r = \frac{2}{5} mr^2 \frac{a}{r^2}$$

$$\Rightarrow\ a = \frac{5}{7}\frac{g\sin\theta}{R - r}$$

$$\omega = \sqrt{\frac{5g}{7(R - r)}}$$

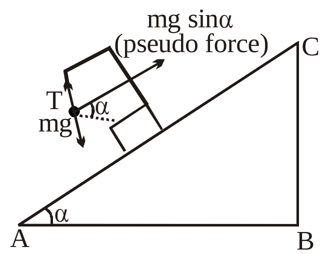
$$\Rightarrow\ T = 2\pi\sqrt{\frac{7(R - r)}{5r}}$$

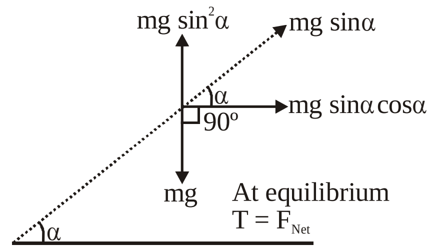
Q.4 (C)

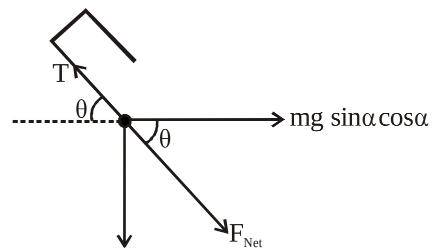
Speed of particle doing SHM decrease as it go away from mean position. Time during which particle remain in extreme position will be longer.

Q.5 (D)

Block slides downwards along the inclined plane with acceleration g sin α.







$mg\,[1 - \sin^2\alpha] \Rightarrow mg\cos^2\alpha$

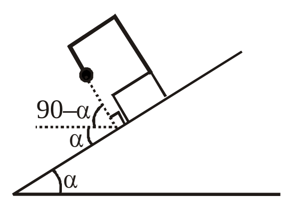
$$\tan\theta = \frac{mg\cos^2\alpha}{mg\sin\alpha\cos\alpha}$$

$\tan\theta = \cot\alpha$

$\tan\theta = \tan(90 - \alpha)$

$\theta = (90 - \alpha)$

string is perpendicular to inclined plane.



Q.6 (C)

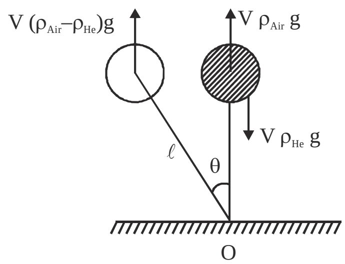
$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(1 + \frac{\theta^2}{16}\right)$$ (This is valid when θ is not small)

$$T_0 = 2\pi\sqrt{\frac{\ell}{g}}$$ (for small θ)

$$T = T_0\left(1 + \frac{\theta^2}{16}\right)$$

$T > T_0$

Q.7 (C)



$\tau_0 = V(\rho_{Air} - \rho_{He})\, g\, \ell \sin\theta$

For small angular displacement (θ)

$\tau_0 = V(\rho_{Air} - \rho_{He})\, g\, \ell\, \theta$

$I\alpha = V[\rho_{Air} - \rho_{He}]\, g\, \ell\, \theta$

$\rho_{He} V \ell^2 \alpha = [\rho_{Air} - \rho_{He}]\, \ell\, \theta\, g$

$$\alpha = \left[\frac{\rho_{Air} - \rho_{He}}{\rho_{He}}\right]\frac{g}{\ell}\theta$$

$$\omega = \sqrt{\left(\frac{\rho_{air} - \rho_{He}}{\rho_{He}}\right)\frac{g}{\ell}}$$

$$= 2\pi\sqrt{\frac{\ell}{g}\left(\frac{\rho_{He}}{g(\rho_{air} - \rho_{He})}\right)}$$

Q.8 (A)

time period will increase as the amplitude is increases

Q.9 (D)

$F = mE_g = ma \Rightarrow a = E_g = \text{constant}$

$$d = \frac{1}{2}at^2 \Rightarrow t \propto dt^{1/2}$$

$$\frac{T_A}{T_B} = \sqrt{2}$$

Q.10 (B)

Given $F = -kx^3$

$$-\frac{dU}{dx} = -kx^3 \qquad \Rightarrow U\ \frac{1}{4}\ kx^4$$

$\therefore$ Energy of oscillations will be

$$E = \frac{1}{2}mv^2 + U = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{4}kx^4 \qquad ...(1)$$

If we pull $\frac{dx}{dt} = 0$ in above equation, we will get amplitude as $A = \sqrt[4]{\frac{4E}{k}}$ ...(2)

Also on rearranging equation (1), we get

$$dt = \pm dx\sqrt{\frac{m}{2E}}\left(1 - \frac{k}{4E}x^4\right)^{-1/2}$$

Now, use $A = \sqrt[4]{\frac{4E}{k}}$, to reduce above equation as

$$dt = \pm dx\sqrt{\frac{2m}{k}}A^{-2}\left(1 - \left(\frac{x}{A}\right)^4\right)^{-1/2}$$

The time period can be found by integrating above equation.

$$T = 4\int_0^A dx\sqrt{\frac{2m}{k}}A^{-2}\left(1 - \left(\frac{x}{A}\right)^4\right)^{-1/2}$$

$$= 4\sqrt{\frac{2m}{k}}A^{-2}\int_0^A\left(1 - \left(\frac{x}{A}\right)^4\right)^{-1/2} dx$$

Put $\frac{x}{A} = u \Rightarrow dx = Adu$

$$\therefore T = 4\sqrt{\frac{2m}{k}}A^{-2}(A)\int_0^1 du\left(1 - u^4\right)^{-1/2}$$

$$T = 4\sqrt{\frac{2m}{k}}A^{-1}(I)$$

Where $I = \int_0^1 (1 - u^4)^{-1/2}\, du$ is a numerical value

So from above equation $T \propto A^{-1}$

$$\therefore \frac{T_2}{T_2} = \frac{2A}{A} \Rightarrow T_2 = \frac{T}{2}$$

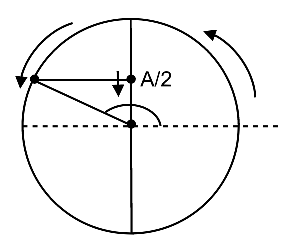
JEE-MAIN PREVIOUS YEAR'S

Q.1 (2)

$K_{eff} = 2K + 2K = 4K$

$$\therefore \quad T = 2\pi\sqrt{\frac{M}{4K}}$$

Q.2 (2)



$$\phi = \frac{\pi}{2} + \frac{\pi}{3}$$

$$\phi = \frac{5\pi}{6}$$

Q.3 (2)

If displaced from equilibrium position,

$$F_{\text{res towards equilibrium}} = \left(\frac{GMm\,r}{R^3}\right)\cos\theta$$

$$= \frac{GMm\,r}{R^3} \cdot \frac{x}{r}$$

$$= \left(\frac{GMm}{R^3}\right)x$$

$$= \left(\frac{gm}{R}\right)x$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{\frac{mg}{R}}} = 2\pi\sqrt{\frac{R}{g}}$$

Q.4 (1)

$$v = \omega\sqrt{A^2 - x^2}$$

$$\frac{v^2}{A^2\omega^2} + \frac{x^2}{A^2} = 1$$

$\therefore$ Ellipse is the correct graph.

Q.5 (1)

Q.6 (1)

Kx = Ma

F + Kx = MaB

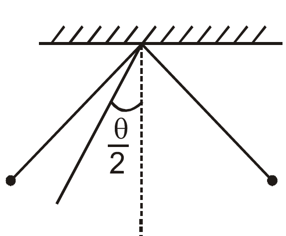
F + Ma = MaB

$$aB = \frac{F}{M} + a$$

Q.7 [7]

Oscillation is $\left(\frac{1}{2} + \frac{1}{8}\right)$

For half oscilation, time required will be $\frac{T}{2}$



$$\theta = \alpha_0 \sin\omega t$$

$$\frac{1}{2}\ \sin\omega t$$

$$\omega t = \frac{\pi}{6}$$

$$\therefore \quad t = \frac{T}{12}$$

$$\therefore \quad \frac{T}{2} + \frac{T}{12} = \frac{7T}{12}$$

Q.8 [3]

$$v = \omega\sqrt{A^2 - x^2}$$

$$\frac{A\omega}{2} = \omega\sqrt{A^2 - x^2}$$

solving we get

$$x = \frac{\sqrt{3}A}{2}$$

Q.9 (1)

$$T = 2\pi\sqrt{\frac{m}{2K}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{2K}{m}}$$

Q.10 [15]

$$\sqrt{\mu_r \varepsilon r} = 2$$

$$v = \frac{C}{n} = \frac{3 \times 10^8}{2} = 15 \times 10^7 \text{ m/s}$$

x = 15

Q.11 (1)

Velocity at mean position is = $A\omega$

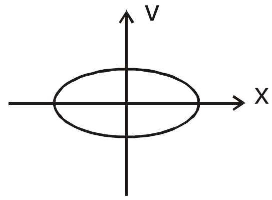
Conserving momentum $MA\omega_O = (M + m)V'$

$$V' = \frac{MA\omega_O}{M + m} = (A')\sqrt{\frac{K}{M + m}}$$

$$A' = \frac{MA\sqrt{\frac{K}{M}}}{M + m} \times \sqrt{\frac{M + m}{K}} = \sqrt{\frac{M}{(M + m)}}A$$

Q.12 (1)

$$v = \omega\sqrt{A^2 - x^2}$$



Q.13 (4)

When lift is stationary

$$T = 2\sum\sqrt{\frac{L}{g}}$$

When lift is moving upwards

$\Rightarrow$ Pseudo force acts downwards

$$\Rightarrow geff = g + \frac{g}{2} = \frac{3g}{2}$$

$\Rightarrow$ New time period

$$T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{2L}{3g}}$$

$$\boxed{T'\sqrt{\frac{2}{3}}T}$$

Q.14 (1)

$$A = A_0 e^{-\gamma t} = A_0 e^{-\frac{bt}{2m}}$$

$$\frac{A_0}{2} = A_0 e^{-\frac{bt}{2m}}$$

$$\frac{bt}{2m} = \ln 2$$

$$t = \frac{2m}{b}\ln 2 = \frac{2 \times 500 \times 0.693}{20}$$

t = 34.65 second.

Q.15 (3)

KE = PE

$$\frac{1}{2}m\omega^2 (A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

$A^2 - x^2 = x^2$

$2x^2 = A^2$

$$x = \pm\frac{A}{\sqrt{2}}$$

Q.16 (2)

$$T_a = 2\pi\sqrt{\frac{M}{K}}$$

$$T_b = 2\pi\sqrt{\frac{M}{K/2}}$$

$$\frac{T_b}{T_a} = \sqrt{2} = \sqrt{x}$$

$\Rightarrow x = 2$

Q.17 (Bonus)

$A = A_0 e^{-\gamma t}$

$$\ln 2 = \frac{b}{2m} \times 120$$

$$\frac{0.693 \times 2 \times 1}{120} = b$$

1.16×10^{-2} kg/sec.

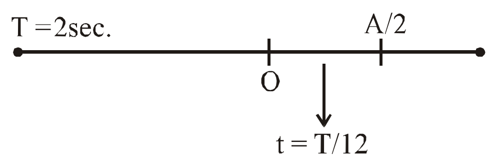
Q.18 (4)

$A_1\omega_1 = A_2\omega_2$

$$A_1\sqrt{\frac{k_1}{m}} = A_2\sqrt{\frac{k_2}{m}}$$

$$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

Q.19 (6)



$$t = \frac{2}{12} = \frac{1}{6}$$

$\therefore$ Correct answer = 6.00

Q.20 (4)

(1) $\sin(\omega t) + \cos(\omega t) = \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$

SHM

$$T = \frac{2\pi}{\omega}$$

(2) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$

Periodic but Non-harmonic

T = LCM of (T_1, T_2, T_3)

$$= \text{LCM of } \left(\frac{2\pi}{\omega}, \frac{2\pi}{2\omega}, \frac{2\pi}{3\omega}\right)$$

$$= \frac{2\pi}{\omega}$$

(3) $\sin^2(\omega t) = \dfrac{1-\cos(2\omega t)}{2} = \dfrac{1}{2} - \dfrac{\cos(2\omega t)}{2}$

Periodic and Harmonic

$T = \dfrac{2\pi}{2\omega} = \dfrac{\pi}{\omega}$

But the given equation does not obey the differential equation of SHM. So it is not an SHM.

(4) $3\cos\left(\dfrac{\pi}{4} - 2\omega t\right)$

SHM

$T = \dfrac{2\pi}{2\omega} = \dfrac{\pi}{\omega}$

Q.21 (1)

Q.22 (4)

$T = 2\pi\sqrt{\ell/g}$

When bob is immersed in liquid

mg_{eff} = mg – Buoyant force (σ = density of liquid)

$= mg - v\dfrac{\rho}{4}g$

$= mg - \dfrac{mg}{4} = \dfrac{3mg}{4}$

$\therefore\ g_{eff} = \dfrac{3g}{4}$

$T_1 = 2\pi\sqrt{\dfrac{\ell_1}{g_{eff}}} \quad \ell_1 = \ell + \dfrac{\ell}{3} = \dfrac{4\ell}{3},\ \ell_{eff} = \dfrac{3g}{4}$

By solving

$T_1 = \dfrac{4}{3}2\pi\sqrt{\ell/g}$

$T_1 = \dfrac{4T}{3}$

Q.23 (4)

Q.24 (4)

Q.25 (4)

Q.26 (2)

Q.27 (6)

Q.28 (10)

Q.29 (2)

Q.30 (4)

Q.31 (1)

Q.32 (2)

Q.33 [1]

Q.34 [2]

Q.35 [8]

Q.36 (4)

Q.37 (3)

Form potential energy curve

$U_{max} = \dfrac{1}{2}kA^2 \Rightarrow 10 = \dfrac{1}{2}k(2)^2$

$\Rightarrow k = 5$

Now $T_{spring} = T_{pendulum}$

$2\pi\sqrt{\dfrac{5}{5}} = 2\pi\sqrt{\dfrac{4}{g}}$

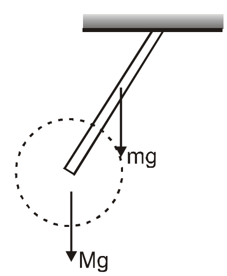
$\Rightarrow 1 = \sqrt{\dfrac{4}{g}} \Rightarrow$ g = 4 on planet

Option (3)

JEE-ADVANCED PREVIOUS YEEAR'S

Q.1 (A), (D)

torque is same for both the cases.

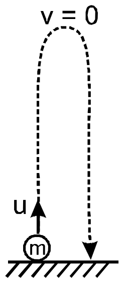


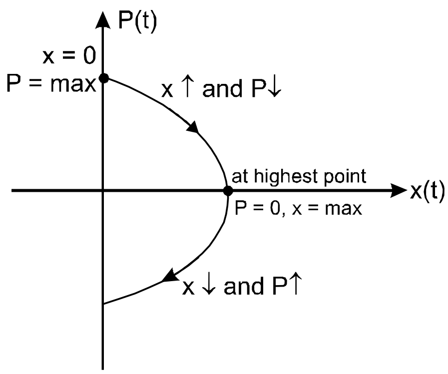
$T = 2\pi\sqrt{\dfrac{I}{mgd}}$

$I_A > I_B$

$\omega_A < \omega_B$

Q.2 (D)





Q.3 (C)

In 1st case amplitude of SHM is a.

In 2nd case amplitude of SHM is 2a

Total energy = $\frac{1}{2}$k(amplitude)^2

$E_1 = \frac{1}{2}k(2a)^2$

$E_2 = \frac{1}{2}k(a)^2$

$E_1 = 4\,E_2$

Alternative :

Linear momentum P = mv

$= m\omega\sqrt{A^2 - x^2}$

$\Rightarrow P^2 = m^2\omega^2(A^2 - x^2)$

$\Rightarrow P^2 + (m\omega)^2x^2 = m^2\omega^2A^2$ ...(i)

Equation of circle (bigger)

$P^2 + x^2 = (2a)^2$

$P^2 + x^2 = 4a^2$ ...(ii)

Equation of circle (smaller)

$P^2 + x^2 = a^2$ ...(iii)

Comparing (i) and (ii)

Amplitude A = 2a

and $(m\omega)^2 = 1 \Rightarrow m\omega^2 = \frac{1}{m}$

$\frac{1}{2}m\omega^2(A)^2$

So energy $E_1 = \frac{1}{2}m\omega^2(2a)^2$

$= \frac{1}{2}\frac{1}{m}\times(4a^2)$

$= \frac{2a^2}{m}$

Comparing (i) and (iii)

A = a

$(m\omega)^2 = 1 \Rightarrow m\omega^2 = \frac{1}{m}$

So $E_2 = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}\times\frac{1}{m}a^2 = \frac{1}{2}\frac{a^2}{m^2}\,\frac{1}{2}\frac{a^2}{m}$

So $\frac{E_1}{E_2} = 4 \Rightarrow E_1 = 4E_2$

Q.4 (B)

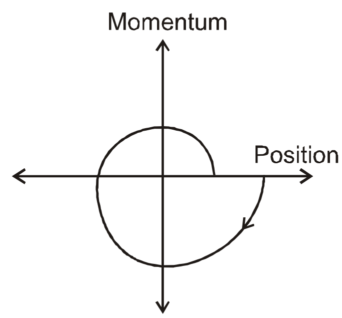
Linear momentum

P = mv

$= m\omega\sqrt{A^2 - x^2}$

$\Rightarrow P^2 + m^2\omega^2x^2 = m^2\omega^2A^2$

represents a circle on P–x diagram with radius of circle R = A ($\because m^2\omega^2 = 1$)



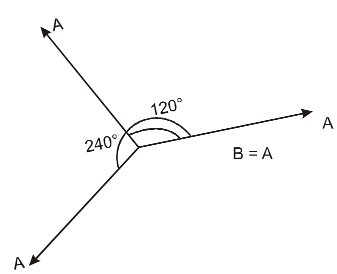
ω of spring mass system remains constant and equal to $\sqrt{\frac{k}{m}}$

Amplitude of oscillation inside liquid will decrease due to viscous force

So radius of circular arcs will decrease as position change

Correctly shown in option B

Q.5 (B)



So $B = A,\ \phi = 240° = \frac{4\pi}{3}$

Q.6 (A)

Time of flight for projectile

$$T = \frac{2u\sin\theta}{g} = 1 \text{ sec.}$$

$$\frac{2u\sin 45}{g} = 1 \text{ sec.}$$

$$u = \frac{g}{\sqrt{2}}$$

$$u = \sqrt{50} \text{ m/s}$$

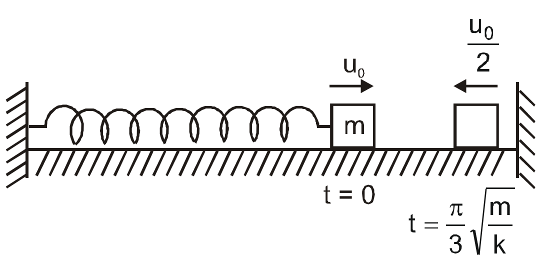
Q.7 (A,D)

$x = A\sin\omega t$

$$v = A\omega\cos\omega t = \frac{\omega A}{2}$$

$$\Rightarrow \cos\omega t = \frac{1}{2}$$

$$\omega t = \frac{\pi}{3} \Rightarrow t = \frac{2\pi}{3} = \frac{\pi}{3}\sqrt{\frac{m}{k}}$$



$$\text{for (C) time} = \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{\pi}{2}\sqrt{\frac{m}{k}}$$

$$= \frac{5\pi}{6}\sqrt{\frac{m}{k}}$$

$$\text{for (D) time} = \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{k}}$$

$$= \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

Q.8 (A,B,D)

Case I : $\omega' = \sqrt{\frac{k}{M+m}}$,

$$\therefore\ MA\sqrt{\frac{k}{M}} = (M+m)A'\sqrt{\frac{k}{M+m}}$$

$$\Rightarrow A' = \sqrt{\frac{M}{M+m}}A$$

Now

$$E' = \frac{1}{2}(M+m)\omega'^2 A'^2 = \frac{1}{2}\frac{kA^2M}{M+m}$$

$$\Rightarrow v' = A\sqrt{\frac{k}{M+m}}$$

Case II : ,amplitude will not change.

$$E' = \frac{1}{2}kA^2$$

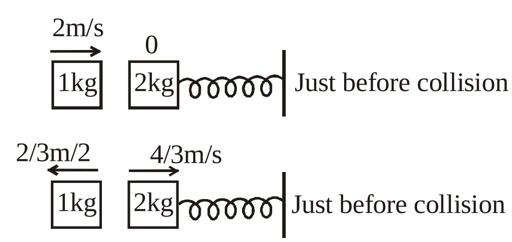
$$\therefore\ v' = A\sqrt{\frac{k}{M+m}}$$

$\therefore$ (a, b, d)

Q.9 [2.09]

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi \sec$$

block returns to original position in $\frac{T}{2} = \pi \sec$



$$d = \frac{2}{3}(\pi) = \frac{2}{3}(3.14) = 2.0933\text{m}$$

$d = 2.09$ m

EXERCISES

ELEMENTARY

Q.1 (4)

Q.2 (3)

Q.3 (3)

Path difference $\Delta = \frac{\lambda}{2\pi} \times \phi \Rightarrow 1 = \frac{\lambda}{2\pi} \times \frac{\pi}{2} \Rightarrow \lambda = 4\text{m}$

Hence $v = n\lambda = 120 \times 4 = 480$ m/s

Q.4 (3)

(3) The given equation representing a wave travelling along –y direction (because '+' sign is given between t term and x term). On comparing it with $x = A \sin(\omega t + ky)$

We get $k = \frac{2\pi}{\lambda} = 12.56 \Rightarrow \lambda = \frac{2 \times 3.14}{12.56} = 0.5$ m

Q.5 (4)

(4) Comparing with standard wave equation

$y = a \sin \frac{2\pi}{\lambda}(vt - x)$,

we get, v = 200 m/s

Q.6 (3)

Q.7 (1)

Q.8 (2)

Q.9 (3)

Q.10 (1)

Q.11 (1)

Q.12 (1)

Q.13 (4)

$$n \propto \frac{1}{l} \Rightarrow \frac{n_2}{n_1} = \frac{l_1}{l_2} \Rightarrow n_2 = \frac{l_1}{l_2} n_1 = \frac{1 \times 256}{1/4} = 1024 \text{Hz}$$

Q.14 (3)

(3) String vibrates in five segment so

$\frac{5}{2}\lambda = l \Rightarrow \lambda = \frac{2l}{5}$

Hence $n = \frac{\nu}{\lambda} = 5 \times \frac{\nu}{2l} = 5 \times \frac{20}{2 \times 10} = 5$ Hz

Q.15 (4)

(4) $n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n \propto \frac{\sqrt{T}}{l}$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{l_2}{l_1}\right)^2 = (2)^2 \left(\frac{3}{4}\right)^2 = \frac{9}{4}$$

Q.16 (3)

(3) $n_1 l_1 = n_2 l_2 \Rightarrow 800 \times 50 = 1000 \times l_2 \Rightarrow l_2 = 40$ cm

Q.17 (2)

(2) Given equation of stationary wave is

$y = \sin 2\pi x \cos 2\pi t$, comparing it with standard equation $y = 2A \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi x}{\lambda}$

We have $\frac{2\pi x}{\lambda} = 2\pi x \Rightarrow \lambda = 1\text{m}$

Minimum distance of string (first mode) $L_{min} = \frac{\lambda}{2} = \frac{1}{2}$ m

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (1)

As $\frac{5\lambda}{2} = 20 \Rightarrow \lambda = 8$ cm

$K = \frac{2\pi}{\lambda} = \frac{314}{4}$

$\omega = KV \; \frac{2\pi}{8 \times 10^{-2}} \times 350 = 27475$

$\therefore y = 0.05 \sin\left(\frac{314}{4}x - 27475t\right)$

Q.2 (3)

$y = \frac{1}{1 + x^2}$ (t = 0)

$y = \frac{1}{1 + (x - 2v)^2}$ (t = 2)

Now comparing

$x - 2v = x - 1$

$v = 0.5\ \frac{m}{sec}$

Q.3 (2)

$V_{P_{max}} = A\omega = Y_0 2\pi f = 4V_\omega$

$Y_0 2\pi f = 4\ \frac{\frac{2\pi f}{2\pi}}{\lambda}\ \therefore\ \lambda = \frac{\pi Y_0}{2}$

Q.4 **(2)**

Put α, β, A, x and t in the equation

$\frac{2\pi}{\lambda} = 0.56\ cm^{-1}$

$2\pi f = 12\ sec^{-1}$

$\alpha x + \beta t + \frac{\pi}{6} = \frac{12.56 \times 180}{3.14} + 30 = 750^o$

$y = 7.5\ cm \sin 750^o = 3.75\ cm.$

$v = \frac{dy}{dt} = A\beta \cos\left(\alpha x + \beta t + \frac{\pi}{6}\right)$

$= 7.5 \times 12 \times \frac{\sqrt{3}}{2} = 77.94\ cm/sec.$

Q.5 (2)

$\omega = 2\pi f = 4\pi\ sec^{-1}$

$K = \frac{2\pi}{\lambda} = 2\pi\ m^{-1}$

$\therefore\ y = 0.5 \cos(2\pi x + 4\pi t)$

Q.6 (3)

Clear from the figure

Q.7 (1)

Standard equation

Q.8 (1)

Solid and in stretched string

Q.9 (1)

Compare with $y = A \sin(\omega t - Kx)$

Q.10 (4)

$y = A \sin(\omega t - kx + \phi)$

at t = 0 and x = 0 $\quad y = -0.5$

$\Rightarrow\ -\frac{1}{2} = \sin\phi \Rightarrow\ \phi = -\frac{\pi}{6}$

Q.11 (4)

$v = \lambda f$

$10 = \lambda 100$

$\lambda = \frac{1}{10}\ m$

$\Delta\phi = \frac{2\pi}{1} \times 10 \times \frac{2.5}{100} = \frac{\pi}{2}$

Q.12 (3)

The length 0.25 m corresponds to 2.5 λ

$0.25 = 0.25\lambda$

$\lambda = 0.1\ m$

$k = \frac{2\pi}{0.1}$

$V = \lambda f$

$330 = 0.1\ f$

$\Rightarrow\ f = 3300$

$\Rightarrow\ \omega = 2\pi \times 3300$

$y = A \sin(\omega t - kx)$

$= 0.25 \sin\left(3300 \times 2\pi t - \frac{2\pi}{0.1} x\right)$

$= 0.25 \sin 2\pi(3300t - 10x)$

Q.13 (4)

Path difference is λ between B and G.

Q.14 (1)

$\lambda = 0.4\ m$

$v = \lambda . f \quad \Rightarrow\ v = 1\ m/s$

Q.15 (3)

$V \propto \sqrt{T}$

$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{2T}{T}} = \sqrt{2}$

Q.16 (1)

$V_{AB} = \sqrt{\frac{6.4g}{10 \times 10^{-3}}} = \sqrt{6400} = 80\ \frac{m}{sec}$

$V_{CD} = \sqrt{\frac{3.2g}{8 \times 10^{-3}}} = \sqrt{4000} = 20\sqrt{10}\ \frac{m}{sec}$

$V_{DE} = \sqrt{\frac{1.6g}{10 \times 10^{-3}}} = \sqrt{1600} = 40\ m/s$

Q.17 (4)

$50 = \sqrt{\frac{mg}{\mu}}$

.........(1)

$52 = \sqrt{\dfrac{m(g^2 + a^2)^{1/2}}{\mu}}$

............(2)

Solve (1) and (2)

$\therefore\ a = 4.1\ \dfrac{m}{sec^2}$

Q.18 (4)

$V = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{YA\alpha\Delta T}{\mu}}$ Put values V = 70 m/s

Q.19 (2)

Distance between boat $= \dfrac{\lambda}{2} = 10$ m

$\Rightarrow\ \lambda = 20m$

time penod, T = 4 sec .

$\therefore\ V = \lambda / T = 20\ m / 4sec.$

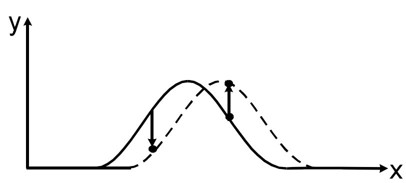
$= 5m/s.$

Q.20 (2)

$R_A = \dfrac{V}{V_A}$, $R_B = \dfrac{V}{V_B}$

as $V_A > V_B$, $R_A < R_B$

Q.21 (2)



Dotted shape shows pulse position after a short time interval. Direction of the velocities are decided according to direction of displacements of the particles.

at x = 1.5 slope is +ve

at x = 2.5 slope is –ve

Q.22 (1)

$y = \dfrac{10}{\pi}\sin\left(200\,\pi\,t - \dfrac{\pi x}{17}\right)$

comparing with

y = A sin (ωt-kx)

$\omega = 2000\pi = \dfrac{2\pi}{T}$

$\Rightarrow\ T = 10^{-3}$ sec

Maximum velocity = Aω

$\Rightarrow\ \dfrac{10}{\pi} \times 2000\pi \times 10^{-2}$

= 200 m/s

Q.23 (4)

$v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{10}{1 \times 10^{-3}}} = 100m/s$

$\therefore\ t = \dfrac{1}{100} = 0.01$ s

Q.24 (3)

Wavelength at lower end is 0.08 m

Velocity of wave on string $= \sqrt{\dfrac{T}{\mu}}$.

$\mu = \dfrac{15}{10} = 1.5$

Tension at lower end $\Rightarrow$ T = 50 N

Tension at upper end $\Rightarrow$ T = 200 N

$v_{lower} = \sqrt{\dfrac{50}{1.5}}$

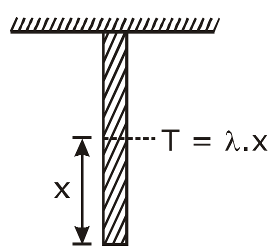
$v_{upper} = \sqrt{\dfrac{200}{1.5}}$

frequency is constant hence $\dfrac{v_1}{\lambda_1} = \dfrac{v_2}{\lambda_2}$

$\dfrac{\sqrt{50}}{0.08} = \dfrac{\sqrt{200}}{\lambda}$

$\Rightarrow\ \lambda = 2 \times 0.08 = 0.16\,m.$

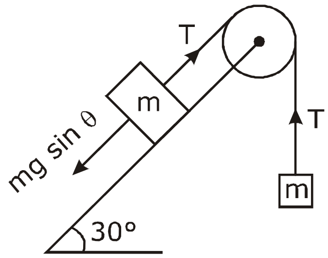
Q.25 (3)



$v = \sqrt{\dfrac{\lambda.x}{\mu}} \Rightarrow v^2 = \dfrac{\lambda}{\mu}.x$

Q.26 (3)

$100 = \sqrt{\dfrac{T}{10^{-2}}} \Rightarrow$ T = 100 N



T = mg sin 30º = 100
m = 20 kg

Q.27 (4)
By defination

Q.28 (3)
As<P> = $2\pi^2 f^2 A^2 \mu v$ put values

$90 = 2 \times 10 \times f^2 \times 25 \times 10^{-4} \times 4 \times 10^{-2} \sqrt{\frac{100}{4\times10^{-2}}}$

$\Rightarrow$ f = 30 Hz

Q.29 (4)

$P \alpha A^2 \quad \frac{P}{0.40} = \frac{4^2}{2^2} \Rightarrow P = 1.6$ watt

Q.30 (1)
By definition

Q.31 (3)
Amplitude varies between $A_1 + A_2$ to $A_1 - A_2$

Q.32 (1)
$A^2_{net} = (y_m + y_m)^2$ for $\phi = o$
$\therefore A_{net} = 2y_m \Rightarrow I_{net} \alpha A^2_{net} \alpha 4y_m^{\ 2} \propto 4I$
where I $\Rightarrow$ Intensity of either wave

Q.33 (4)
$I = 2\pi^2 A^2 f^2 \rho V$
$I = 2 (3.14)^2 (10^{-8})^2 \times (2 \times 10^3)^2 (1.3) (340)$
$= 3.5 \times 10^{-6}$ wm^{-2}

Q.34 (4)
Resultant wave amplitude $\rightarrow$ depends on phase difference
If p.d = $2n\pi \Rightarrow A_{max} = 2A$
p.d = $(2n + 1)\pi \Rightarrow A_{max} = 0$

Q.35 (3)
Resultant amplitude depends on ϕ

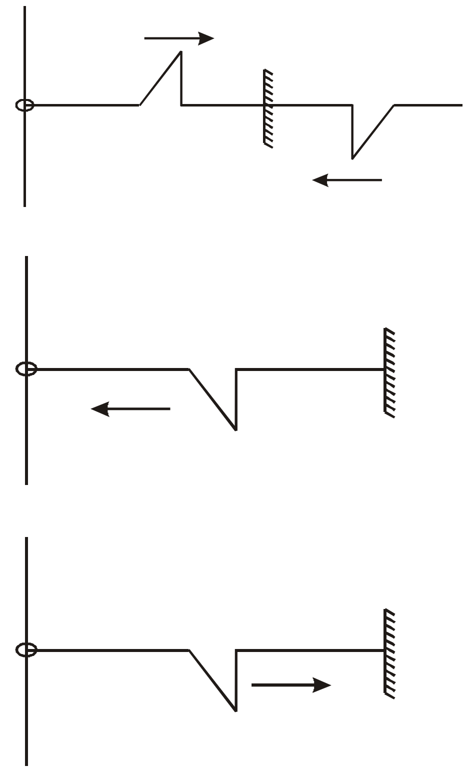
$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$

Frequency of the resulting wave is same.

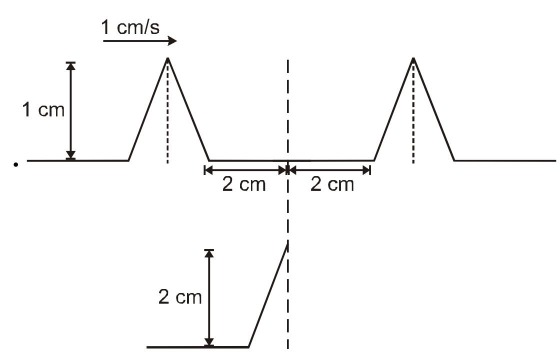
Q.36 (4)
The waves will pass through each other without any change in their shape.

Q.37 (3)
Second string is denser so speed will decreases.

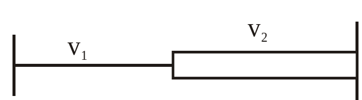
Q.38 (1)



Q.39 (4)



Q.40 (3)
$V_2 < V_1$



$\Rightarrow \lambda' < \lambda$

Q.41 (2)
As x = 0 is node $\Rightarrow$ standing wave should be $y = 2a \sin kx \sin \omega t$
Now solve

Q.42 (3)

$$\frac{n}{2\ell}\sqrt{\frac{T}{\mu}} = 350 \text{ and } \frac{n+1}{2\ell}\sqrt{\frac{T}{\mu}} = 420$$

$$\therefore \quad \frac{n}{n+1} = \frac{350}{420} \Rightarrow n = 5 \therefore \frac{5\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\lambda}{5}$$

$$\frac{v}{f} = \frac{2\ell}{5} \Rightarrow \frac{v}{2\ell} = \frac{f}{5} \Rightarrow f' = \frac{f}{5} = 70 \text{ Hz}$$

Q.43 (4)

At $x_1 = \frac{\pi}{3K}$ and $x_2 = \frac{3\pi}{2K}$

Nodes are not formed because neither x_1 nor x_2 gives $\sin kx = 0$

$$\therefore \Delta x = x_2 - x_1 = \frac{7\pi}{6K}$$

As this Δx is between λ and $\frac{\lambda}{2}$

$$\therefore \phi_1 = \pi$$

and $\phi_2 = K\Delta x = \frac{7\pi}{6}$

$$\therefore \frac{\phi_1}{\phi_2} = \frac{6}{7}$$

Q.44 (2)

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{A\rho}} = \sqrt{\frac{T}{\pi r^2 \rho}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}\cdot\frac{\rho_2}{\rho_1}\cdot\frac{r_2^2}{r_1^2}} = \sqrt{2\times\frac{1}{2}\times\frac{1}{4}} = \frac{1}{2}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}\cdot\frac{\rho_2}{\rho_1}\cdot\frac{r_2^2}{r_1^2}} = \sqrt{2\times\frac{1}{2}\times\frac{1}{4}} = \frac{1}{2}$$

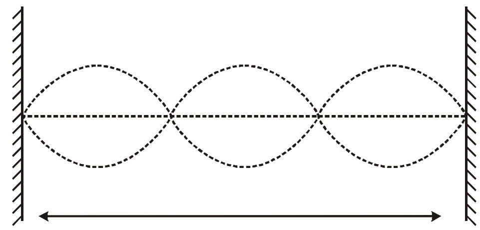
Q.45 (3)

$$f = \frac{4V}{2L} \Rightarrow \lambda = \frac{v}{f} = \frac{L}{2} = 40 \text{ cm}$$

Q.46 (1)
Satisfy the standard equation of wave.

Q.47 (2)

$$\therefore \quad v = \frac{2}{3}\times 300 = 200 \text{ m/s}$$



$$L = \frac{3\lambda}{2} = 1$$

$$\Rightarrow \quad \lambda = \frac{2}{3}$$

Q.48 (2)
$y = 10 \sin 2\pi (100\, t - 0.02\, x) + 10 \sin 2\pi (100\, t + 0.02x)$
$= 2 \times 10 \sin 200\, \pi t \cos 2\pi (0.02)x$
$200\, \pi = \omega$

$$2\pi \times 0.002 = \frac{2\pi}{\lambda}$$

loop length = $\frac{\lambda}{2}$

Q.49 (3)
$\mu = 9\times10^{-3}$ kg
$T = 360$ N

$$\text{frequency} = \frac{nv}{2L}$$

Hence $\frac{nv}{2L} = 210$

$$\frac{(n+1)v}{2L} = 280$$

$$\frac{n}{n+1} = \frac{21}{28} \quad \Rightarrow \quad 28n = 21x + 21$$

$3n = 21$
$n = 3$
In first case

$$\lambda = \frac{2L}{3}$$

$$L = \frac{3\lambda}{2}$$

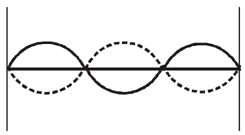
A loop is formed in length $\frac{\lambda}{2}$
Hence no. of loops are 3

Q.50 (1)
y = a cos kx sin ωt
compare it with
y = 2A cos kx sin ωt

$A = \frac{a}{2}$ (Amplitude of each wave)

Q.51 (4)

$L = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2L}{3}$



$\therefore \quad \omega = 2\pi f = \frac{2\pi V.3}{2L} = \frac{3\pi v}{L}$

x = 0 is node

$\Rightarrow$ amplitude = A sin kx = $A \sin \frac{3\pi x}{L}$

$y(x, t) = A \sin\left(\frac{3\pi x}{L}\right)\cos\left(\frac{3\pi v\ell}{L}\right)$

Q.52 (2)

$\frac{\lambda_1}{2} = 2 \text{ cm} \Rightarrow \lambda_1 = 4 \text{ cm}$

$\frac{\lambda_2}{2} = 1.6 \text{ cm} \Rightarrow \lambda_2 = 3.2 \text{ cm}$

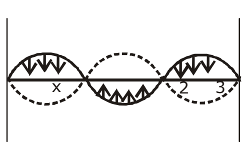
$f_1 = \frac{n}{2\ell} v_w = \frac{v_w}{\lambda_1} \Rightarrow \frac{1}{\lambda_1} = \frac{n}{2\ell}$

$f_2 = \frac{(n+1)}{2\ell} v_w = \frac{v_w}{\lambda_2} \Rightarrow \frac{1}{\lambda_2} = \frac{(n+1)}{2\ell}$

$\Rightarrow \frac{1}{\lambda_2} - \frac{1}{\lambda_1} = \frac{1}{2\ell}$

$\Rightarrow \ell = \frac{\lambda_1 \times \lambda_2}{\lambda_1 - \lambda_2}$ 8 cm

Q.53 (3)



Q.54 (3)

$y = 0.02 \cos(10\pi x). \cos(50\pi t + \frac{\pi}{2})$

Now,
$\omega = 50\pi = 2\pi f$

$f = 25$ Hz and $10\pi = \frac{2\pi}{\lambda}$

$\Rightarrow \lambda = \frac{1}{5}$ m

For antinode
$\cos(10\pi x) = \pm 1$
For Node $\cos(10\pi x) = 0$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (A)
$Z = e^{-(x+2)2}$ $z = e^{-(2-x)2}$
t = 0 t = 1
General equation
$z = = e^{-(x-vt+2)2}$
at t = 1s.
$-vt + 2 = -2$
and positive x direction
$\therefore$ v = 4 m/s

Q.2 (C)

$v = f\lambda = \frac{54}{60} \times 10$

= 9 m/sec.

Q.3 (A)

As $f_1 = f$, $f_2 = \frac{f}{2}$, $f_3 = f$

$\therefore \omega_1 = 2\pi f \Rightarrow \omega_3 = 2\pi f$
and $\omega_2 = \pi f$

Q.4 (C)
Satisfy the standard equation of wave

Q.5 (A)

$\mu = \frac{m}{\ell} = \rho A$

$\therefore m_1 = \rho\pi r^2$
$m_2 = \rho 4\pi r^2$

$\therefore \frac{v_1}{v_2} = \frac{\sqrt{T/\mu_1}}{\sqrt{T/\mu_2}}$

Let P loops and q loops are formed respectively 1^{st} and 2^{nd} ωire.

$$\therefore \quad \frac{p}{2\ell} V_1 = \frac{q}{2\ell} V_2 \qquad \Rightarrow \frac{p}{q} = \frac{1}{2}$$

Q.6 (A)

If $T = mg = v\rho g$

$$\therefore \quad f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = 300$$

.....(1)

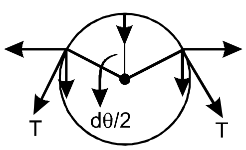
Now $T' = mg - f_b = v\rho g - \frac{V}{2}\,g.$

$$T' = vg\left(\frac{2\rho - 1}{2}\right)$$

$$\therefore \quad f' = \frac{1}{2\ell}\sqrt{\frac{vg}{\mu}\frac{(2\rho-1)}{2}} \quad ....(2)$$

$$\therefore \quad \frac{f'}{f} = \left(\frac{2\rho-1}{2\rho}\right)^{1/2} f' = 300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$$

Q.7 (a) (B), (b) (C), (c) (D)

(a) 

$$2T\sin\frac{d\theta}{2} = dm.\omega^2 R$$

$$2T\frac{d\theta}{2} = \frac{m}{\ell} R d\theta.\omega^2 R \quad T = \frac{m\omega^2 R^2}{\ell}$$

$$\therefore \quad V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\frac{m}{\ell}}} = \omega R^2 \;\therefore\; V = \frac{\omega L}{2\pi}$$

(b) $V_{P/R} = \frac{\omega L}{2\pi}$

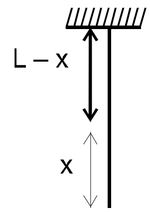
$$V_{P/G} - V_{R/G} = \frac{\omega L}{2\pi} \qquad \Rightarrow V_{P/G} - \frac{\omega L}{2\pi} = \frac{\omega L}{2\pi}$$

$$\therefore \quad V_{P/G} = \left(\frac{\omega L}{2\pi}\right) 2\; V_{P/G} = \frac{\omega L}{\pi}$$

(c) $V_{P/R} = \frac{\omega L}{\pi} \; V_{P/G} - \left(-\frac{\omega L}{2\pi}\right) = \frac{\omega L}{2\pi}$

$\therefore \quad V_{P/G} = 0$

Q.8 (B)



For the pulse

$$V = \sqrt{\frac{\mu x g}{\mu}} = \sqrt{xg} = \frac{dx}{dt}$$

$$\frac{dx}{dt} = \sqrt{xg} \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \sqrt{g}\int_0^t dt$$

$$t = 2\sqrt{\frac{x}{g}}$$

————(1)

for the particle

$$L - x = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2(L-x)}{g}}$$

————(2)

from equation (1) & (2)

$\Rightarrow \;\therefore\; x = \frac{L}{3}$ from the bottom

Q.9 (C)

$$V_{max} = A\omega = 5 \;\Rightarrow A\frac{2\pi}{4} = 5 \Rightarrow \quad A = \frac{10}{\pi}\text{ cm.}$$

Q.10 (D)

$$V_{\omega} = \frac{\omega}{k} = \frac{420}{21} = 20$$

$$\therefore V = \sqrt{\frac{T}{\mu}} = 20$$

$\Rightarrow \quad T = (20)^2\mu = (20)^2 \times 0.2 = 80N$

Q.11 (B)

As, $T = \frac{YA}{20}\,4 \Rightarrow \nu_0 = \frac{1}{2\times 24}\sqrt{\frac{YA}{20}.\frac{4}{\mu}}$ ....(1)

and $T' = \frac{YA}{20}\,6 \Rightarrow \nu' = \frac{1}{2\times 26}\sqrt{\frac{YA}{20}.\frac{6}{\mu}}$ ....(2)

$$\therefore \quad \frac{\nu'}{\nu_0} = \frac{24}{26}\sqrt{\frac{6}{4}}$$

$\therefore \quad \nu' > \nu_0$

Q.12 (A)

$P_{avg} = 2\pi^2A^2f^2\mu v$

Initially $P_0 = 2\pi^2A^2f_0^{\,2}\mu v$ ... (1)

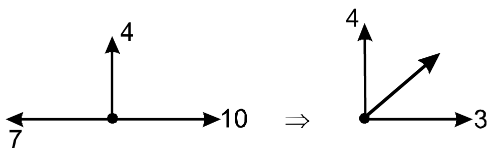
as frequency depends only on source. So, it is not change then

$\frac{P_0}{2} = 2\pi^2A^2f_0^{\,2}\mu v$ ... (2)

divide (1)/(2) $\Rightarrow 2 = \frac{A_0^2}{A^2}$

$A = \frac{A_0}{\sqrt{2}}$

Q.13 (C)



Resultant Amplitude $= \sqrt{3^2+4^2} = 5\mu m$

Q.14 (C)

As y = A sin (Kx – ωt + 30º) for incident wave

Now for reflected wave : Energy α Amp²

$\therefore$ Y = 0.8 A sin (–Kx – ωt + 30 + 180)

Y = 0.8 A sin (–Kx – ωt + 210)

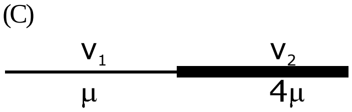
Y = –0.8 A sin (kx + ωt –210)

Y = –0.8 A sin [Kx + ωt – 30 – 180]

Y = 0.8 A sin [180 – (Kx + ωt – 30)]

Y = 0.8 A sin (Kx + ωt – 30)

Q.15 (C)



y = 6 sin (5t + 40x)

$v_1 = \sqrt{\frac{T}{\mu}}$

$v_2 = \sqrt{\frac{T}{4\mu}} = \frac{v_1}{2} = \left(\frac{v_1/2 - v_1}{3v_1/2}\right)6 = 2$ mm.

$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) = 4$ mm

Reflected there will be phase difference of π.

y = – 2 sin (5t – 40x)

Q.16 (B)

$P = 2\pi^2A^2f^2\mu v \quad P_i = 2\pi^2(6)^2f\mu v_1$

$= \underbrace{2\pi^2 f\mu v_1 .36}_{K_1}$

$P_r = 2\pi^2(2)^2f^2\mu.v_1$

$P_t = 2\pi^2(4)^2f^2.\ \frac{4\mu v_1}{2} = K_1 \times 32$

$\therefore \quad \% = \frac{K_1 \times 32}{K_1 \times 36} \times 100 \approx 89\%$

Q.17 (C)

Given $v = \sqrt{\frac{T}{\mu}}$, $T_2 = T_1$ and

$\mu_2 = 4\mu_1$

then $v_2 = \frac{v_1}{2}$

Frequency doesn't change that is

$\omega_1 = \omega_2$

Then

$K_2 = \frac{\omega_2}{v_2} = \frac{\omega_1}{v_1/2} = 2K_1$

$A_t = \left(\frac{2v_2}{v_1+v_2}\right)A_i = \frac{2v_1/2}{v_1 + v_1/2}A_i = \frac{2}{3}A_i$

Then equation of transmitted wave

$y_t = \frac{2}{3}A_i \cos(2k_1x - \omega_1 t)$

Q.18 (C)

$f_1 = \frac{1}{2\ell_1}\sqrt{\frac{T\ell_1}{m}} \qquad f_2 = \frac{1}{2\ell_2}\sqrt{\frac{T\ell_2}{m}}$

$\therefore \frac{f_1}{f_2} = \sqrt{\frac{\ell_2}{\ell_1}} = \sqrt{\frac{\ell_1(1-20\alpha)}{\ell_1}}$

$\frac{f_1}{f_2} = (1 - 10\alpha)$ (By Binomial theorem)

$\alpha = \frac{f_2 - f_1}{10f_2} = 10^{-4}\ ^\circ C^{-1}$

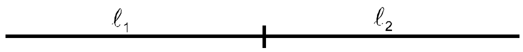
Q.19 (D)

$n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \quad n = \frac{1}{2\ell}\sqrt{\frac{YA\alpha\,\Delta T}{\mu}}$

Put values

$\therefore$ n = 11 Hz

Q.20 (C)



$n_1 = \frac{v}{2\ell_1}$ $n_2 = \frac{v}{2\ell_2}$ and $n = \frac{v}{2\ell}$ (for complete length of wire)

As $\ell = \ell_1 + \ell_2 + \ldots$

$$\frac{V}{2n} = \frac{V}{2n_1} + \frac{V}{2n_2} + \ldots$$

$$\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \ldots$$

Q.21 (B)

$$f = \frac{1}{2\times 40}\sqrt{\frac{mg}{\mu}} = 256 \quad \ldots\ldots(1)$$

$$f = \frac{1}{2\times 22}\sqrt{\frac{mg - f_b}{\mu}} = 256$$

......(2)

$$\frac{mg - f_b}{mg} = \left(\frac{22}{40}\right)^2$$

$$\frac{v(1)g}{v\sigma g} = \frac{40^2 - 22^2}{40^2}$$

$$\sigma = \frac{40^2}{40^2 - 22^2}$$

Q.22 (B)

$\frac{\omega}{K} = V_{\omega}$ for either component waves

Q.23 (C)

$K = 0.025\,\pi = \frac{2\pi}{\lambda}$ $\lambda = \frac{2cm}{0.025}$

Required length $= \frac{\lambda}{2} = \frac{1}{0.025} = 40$ cm

Q.24 (A)

$$\frac{n}{2\ell}\sqrt{\frac{T}{\mu}} = 384 \quad \frac{n-1}{2\ell}\sqrt{\frac{T}{\mu}} = 288$$

$$\therefore \quad \frac{n}{n-1} = \frac{4}{3}$$

$\therefore$ n = 4 Now ; $4\left(\frac{V}{2L}\right) = 384$

Put L = 75 cm

$\therefore$ V = 144 m/sec.

Q.25 (C)

For third overtone $\frac{4\lambda}{2} = \ell$

$\Rightarrow 2\lambda = \ell \Rightarrow \lambda = \frac{\ell}{2}$

As x = 0 is a node

$\therefore A_{s.\omega.} = A \sin \frac{2\pi}{\lambda} x$

$$= a \sin\left(\frac{2\pi}{\frac{\lambda}{2}}\frac{1}{3}\right) = a\frac{\sqrt{3}}{2}$$

Q.26 (C)

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{4.5\times 10^7}{0.05}}$$

$$\frac{n}{2\ell}\sqrt{\frac{4.5\times 10^7}{0.05}} = 420$$

———(1)

$$\frac{n+1}{2\ell}\sqrt{\frac{4.5\times 10^7}{0.05}} = 490 \text{ ———(2)}$$

From equation (1) & (2) $\Rightarrow \frac{n}{n+1} = \frac{6}{7} \Rightarrow n = 6$

Put n in (1) $\therefore \frac{6}{2\ell}\, 3\times 10^2 = 420$

$$\ell = \frac{30000}{140} \quad \ell = \frac{1500}{7} = 214 \text{ cm}$$

Q.27 (A)

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\times 1}\sqrt{\frac{\frac{YA}{\ell}.\Delta\ell}{\rho A}}$$

$$= \frac{1}{2}\sqrt{\frac{9\times 10^{10}\times 5\times 10^{-14}}{9\times 10^3}} = 35 \text{ Hz}$$

Q.28 (B)

In wire B tension T_B, density ρ_B

radius = r

So, $M_B = \pi r^2_B \ell\rho$

$\Rightarrow \mu_B = \pi r^2_B \rho_B$

In wire A $T_A = 2T_B$

$\rho_A = 2\rho_B \; r_A = 2r_B$
$m_A = \pi(2r_B)^2\ell\rho_A = 8\pi r^2{}_B\rho_B\ell$
$\mu_A = 8\pi r^2{}_B\rho_B = 8\mu_B$

$$f_A = \frac{1}{2\ell}\sqrt{\frac{T_A}{\mu_A}} = \frac{1}{2\ell}\sqrt{\frac{2T_B}{8\mu_B}} = \frac{1}{2\ell}\left(\frac{1}{2}\right)\sqrt{\frac{I_B}{\mu_B}}$$

$$f_B = \frac{1}{2\ell}\sqrt{\frac{T_B}{\mu_B}} \Rightarrow f_A : f_B = 1 : 2$$

Q.29 (D)

$f = \frac{v}{\ell} \propto \sqrt{T}$, $f = K\sqrt{T}$ in water

$\frac{f}{2} = K\sqrt{T_1}$
$T = mg$
$T_1 = T/4$
$mg - v\rho_w g = mg/4$

$v.\rho_w.g = \frac{3mg}{4}$ ... (1)

$v = \frac{2}{3} \times 300 = 200$ m/s

$f/3 = K\sqrt{T_2}$
$T_2 = T/9$
$mg - v\rho_\ell.g = mg/9$

$v\rho_\ell g = \frac{8mg}{9}$ ... (2)

(2)/(1) $\frac{\rho_\ell}{\rho_w} = \frac{8\times 4}{9\times 3} = \frac{32}{27}$

Q.30 (A)

$f = \frac{(n+1)}{2\ell}v$

$\Rightarrow \frac{v}{\lambda} = \left(\frac{n+1}{2\ell}\right)v$

$\Rightarrow \lambda = \frac{2\ell}{(n+1)}$

$\Rightarrow d = \frac{\lambda}{4} = \frac{\ell}{2(n+1)}$

$\Rightarrow 2d(n+1) = \ell$

Q.31 (B)

$y = \frac{T/A}{\ell\alpha\frac{\Delta T}{\ell}} \Rightarrow \frac{T}{A} = y.\alpha\,\Delta t$

$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{y.\alpha\Delta T.A}{\rho A}}$

$f = \frac{1}{2\ell}\sqrt{\frac{y.\alpha\Delta T}{\rho}}$

$f \propto \sqrt{\frac{y\alpha}{\rho}}$

Q.32 (C)

$y = A\sin\left(\frac{20}{3}\pi x\right)\cos(1000\pi t)$

$A\sin\left(\frac{20}{3}\pi x\right) = \frac{A}{2}$

$\frac{20}{3}\pi x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = \frac{1}{40}, \frac{5}{40}, \frac{7}{40}$

$x = \frac{1}{20} = 5$ cm.

Q.33 (D)

$v = \sqrt{\frac{100}{0.01}} = 100$ m/s

$f = \frac{n}{2\ell}v$

$f_1 = 50$ Hz 100 Hz 150 Hz
$f_1 \; f_2 \; f_3$

for open $f = \frac{(2n+1)}{4\ell}v$

$f_1 = 25\,H_2, 75\,H_2, 125\,H_2$
$n_1 \; n_2 \; n_3$

Q.34 (B)
Given $y_1 = a\cos(kx - \omega t)$
if x = 0 is a node there amplitude of standing wave be have
$A = 2A\sin kx$
$y_2 = -a\cos(kx + \omega t)$
$y_1 y_2 = a[\cos(kx - \omega t) - \cos(kx + \omega t)$
$= -a\cos(kx + \omega t)$

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A, D)
Compare with $y = A\cos(\omega t - kx)$

Q.2 (A, B, C, D)
Compare with $y = a\sin(\omega t + Kx)$

Q.3 (A, C)

$V_{max} = A\omega = \frac{v}{10} \Rightarrow 10^{-3}\,2\pi f = \frac{10}{10}$

$\Rightarrow f = \frac{1000}{2\pi}$ Hz

Now $V = f\lambda$ find λ

Q.4 (A,B,C,D)

Given that

$y = 10^{-4} \sin(60t + 2x)$

compare it with $y = A \sin(\omega t + kx)$

(a) $v = \frac{\omega}{k} = \frac{60}{2} = 30$ m/s –ve direction

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$

(c) $f = \frac{\omega}{2\pi} = \frac{30}{\pi}$ Hz

(d) $A = 10^{-4}$ m

Q.5 (C)

for x = 4 m

$y = 2\sin(8\pi - 100\pi t + \frac{\pi}{3})$

$= -2\sin(100\pi t - \frac{\pi}{3})$

$\therefore \quad 0 = -2\sin(100\pi t - \frac{\pi}{3})$

$t = \frac{1}{300}$ s.

Q.6 (A,B,D)

$y = \cos(500t - 70x)$

$v = \frac{500}{70} = \frac{50}{7}$ m/s

$f = \frac{\omega}{2\pi} = \frac{500}{2\pi} = \frac{250}{\pi}$

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{70} \times 10^{-2}$ cm

$\lambda = \frac{20\pi}{7}$ cm.

Q.7 (C,D)

$\frac{\partial y}{\partial t} = -v\frac{\partial y}{\partial x}$

Q.8 (B, D)

$v = \sqrt{\frac{T}{\mu}}$

$v = \sqrt{xg}$

$a = \frac{vdv}{dx} = \sqrt{xg} \times \left(\frac{1}{2}\sqrt{\frac{g}{x}}\right) = \frac{g}{2}$ = constant

Q.9 (B,C)

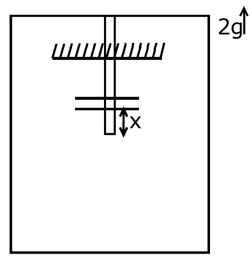
$\lambda = Ax + B$

at

$x = 0, \lambda = 0$

$\therefore \quad B = 0$

So, $\lambda = Ax$



[at x = L]

$\lambda_0 = AL$

$\Rightarrow \quad A = \frac{\lambda_0}{L}$

$\therefore \quad \lambda = \frac{\lambda_0}{L} x$

$\therefore \quad v = \sqrt{\frac{\frac{\lambda_0}{L} \cdot \frac{x^2}{2g}}{\frac{\lambda_0}{L}.x}} = \sqrt{\frac{x}{2g3}} \left\{a = \frac{vdv}{dx} = \frac{3g}{4}\right\}$

$T = \int_0^x \frac{\lambda_0}{L} x \; dx \;\; 3g$

$T = \frac{\lambda_0}{L}\frac{x^2}{2g \times 3}$

Q.10 (A,B)

$y = 2\sin(\pi x)\sin(100\pi t)$

$v_p = \frac{\partial y}{\partial t} = 2 \times 1000\pi \; \sin(\pi x)\cos(100\pi t)$

(a) Max. Displacement

$= 2\sin\left(\pi \times \frac{1}{6}\right)$

$= 2 \times \frac{1}{2} = 1$ mm

(b) Velocity of the particle at x = $\frac{1}{6}$ cm ; t = $\frac{1}{600}$ sec.

v = 2 × 1000 π sin $\frac{\pi}{6}$ cos $\left(100\pi \times \frac{1}{600}\right)$

$= 157\sqrt{3}\ \text{cm/sec}$

(c) k = π ⇒ λ = 2 cm

5 loop = $\frac{5\lambda}{2}$ = 5 cm.

Q.11 (A,D)

$$A_t = \left[\frac{2v_2}{v_1 + v_2}\right] A_i$$

The phase of transmitted wave is same as that of incident wave.

$$A_r = \left[\frac{v_2 - v_1}{v_1 + v_2}\right] A_i$$

Phase of reflected wave depends upon v_1 and v_2.

Q.12 (C,D)

A_{min} = |3A – A| = 2A

A_{max} = A + 3A = 4 A

Distance between maximum and adjacent minima =

$$\frac{\lambda}{4} = \frac{1}{4}\left(\frac{v}{f}\right)$$

Q.13 (A, B, C, D)

Compare with y = A cos $\left(\omega t + \frac{\pi}{2}\right)$ cos (kx)

i.e. y = A sin ωt cos Kx

ω = 50 π ,k = 10 π , v = 5 m/s

$$\frac{2\pi}{\lambda} = 10\,\pi$$

λ = $\frac{1}{5}$ = 0.2 m Position of antinode = 0, $\frac{\lambda}{2}$, λ , $\frac{3\lambda}{2}$, 2λ

= 0 , 0.1, 0.2, 0.3

position of node = 0.05, 0.15, 0.25,

Q.14 (A,B,C,D)

Compare with y = A sin Kx cos ωt

Q.15 (A, D)

Use principle of superposition

Q.16 (A, C)

$V \propto \sqrt{T}$

$f \propto \sqrt{T}$

$$\frac{f}{f+15} = \left(\frac{T}{T + \frac{21T}{100}}\right)^{½} = \frac{10}{11}$$

11f = 10f + 150

f = 150 Hz

Q.17 (C, D)

$$V_w = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{T}{\rho\pi r^2}}$$

$$V'_w = \sqrt{\frac{2T}{2\rho\pi(2r)^2}} = \frac{1}{2}V_w$$

$$f'_A = \frac{v_w}{2L}$$

$$f' = \frac{n}{2(2\ell)} \times v_w' = \frac{n}{4} f_A = \text{3rd overtone}$$

Q.18 (A, C)

$$\int_0^L \frac{1}{2}\mu.dx\left(2A\sin kx\right)^2$$

$\propto \mu A^2 \omega^2 L$

$\propto \mu A^2 (2\pi)^2 f^2 L$

$\propto n^2$

Q.19 (A,D)

Q.20 (C)

Q.21 (B,C)

Q.22 (C, D)

$$\frac{\partial y}{\partial t} = -v_w \frac{\partial y}{\partial x}$$

v_w = Negative

Q.23 (D)

Q.24 (B)

Q.25 (B)

(23-25)

Medium of string BC is denser. These will be π phase difference between reflected and transmitted wave.

$$\frac{\lambda_r}{\lambda_t} = \frac{v_1}{v_2} = \sqrt{\frac{\mu_2}{\mu_1}}$$

Now $\frac{\mu_1}{\mu_2} = \frac{4}{9}$

$$\frac{A_r}{A_t} = \frac{\frac{v_2 - v_1}{v_1 + v_2}Ai}{\frac{2v_2}{v_1 + v_2}.Ai} = \frac{v_2 - v_1}{2v_2} = \frac{1}{4}$$

Q.26 (A) R (B) P (C) S (D) Q

Use x = 0; t = 0 for y and particle velocity $\frac{\partial y}{\partial t}$. Like for (a), y = 0 at x = 0 and t = 0. $\frac{\partial y}{\partial t}$ > 0 i.e. positive therefore it matches with (R).

Q.27 (A) p,q,s (B) s (C) q,r,s (D) s,t

(A) Number of loops (of length $\lambda/2$) will be even or odd and node or antinode will respectively be formed at the middle.

Phase of difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

(B) and (D) Number of loops will not be integral. Hence neither a node nor an antinode will be formed in in the middle.

Phase of difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

A pulse is started at a time t = 0 along the +x direction on a long, taut string. The shape of the pulse at t = 0 is given by function f(x) with

$$f(x) = \begin{cases} \frac{x}{4} + 1 & \text{for} \quad -4 < x \le 0 \\ -x + 1 & \text{for} \quad 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

here f and x are in centimeters. The linear mass density of the string is 50 g/m and it is under a tension of 5N,

NUMERICAL VALUE BASED

Q.1 [0500]

(a) Given $\frac{P_{Fe}}{P_{Al}} = \frac{\sqrt{\frac{T}{\rho_{Al}}}}{\sqrt{\frac{T}{\rho_{Fe}}}} = \sqrt{\frac{7.5}{2.7}} = \frac{5}{3}$

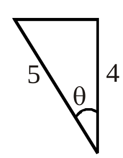
Here fifth harmonic of Fe = third harmonic of Al wire.

(b) Using $P_{Fe} = 5$; f =

$$\frac{5}{2\times1}\sqrt{\frac{75\pi\times4}{3.14\times10^{-6}\times7.5\times10^{3}}} = 500 \text{ Hz}$$

Q.2 [60 Hz]

Q.3 [25 Hz]



$\cos\theta = \frac{4}{5}$

mg = 2T cosθ

$T = \frac{mg}{2\cos\theta}$, $v = \sqrt{\frac{T}{\mu}}$ = 100 m/s,

$$\frac{3\lambda}{2} = 6\text{m}$$

$\lambda = 4$m

$$v = \frac{3}{2L}\sqrt{\frac{T}{\mu}} = 25 \text{ Hg}$$

Q.4 [25 kg]

$$\ell = \frac{5\lambda_1}{2} \Rightarrow \lambda_1 = \frac{2\ell}{5}$$

Now, $f = \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}}$ ....(1)

$f = \frac{3}{2l}\sqrt{\frac{mg}{\mu}}$ ....(2)

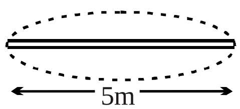
$\sqrt{9} \times 5 = \sqrt{m} \times 3$

$\sqrt{m} = 5 \Rightarrow m = 25$ kg

Q.5 [22 Hz]

Q.6 [4 Hz]

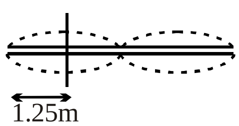
When student stands at middle



$\frac{\lambda}{2} = 5m$, $\lambda = 10$ m

wave velocity $v = \nu\lambda = 20$ m/s

when student stands at 1.25 m



$\lambda/4 = 1.25$ m ; $\lambda = 5$m

$\nu = \frac{v}{\lambda} = \frac{20}{5} = 4$ Hz

Q.7 [0007]

$340 - V_m = \frac{3015}{9}$

$340 + V_T = \frac{855}{5} \times 2 = 342$

$V_T = 2$ m/s, $V_m = 5$ m/s

$V_{rel} = 7$ m/s

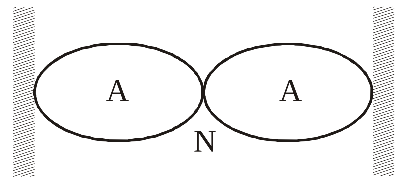
Q.8 [22]

$\Delta L = L_0 \cdot a \cdot \Delta T = -(1 \times 1.21 \times 10^{-5} \times 20)$

$= -2.42 \times 10^{-4}$

$\Rightarrow$ Strain $= 2.42 \times 10^{-4}$

Stress = Young's modulus × Strain



Tension = Stress × Area

$= (2 \times 10^{11}) \cdot (2.42 \times 10^{-4}) \cdot (10^{-6})$ N

$= 48.4$ N

The vibration here

The point pluckes would be an anti-node. As can be seen, the string will vibrate in its second harmonci.

$v = \frac{2}{2L}\sqrt{\frac{T}{M}} = \sqrt{\frac{48.4}{0.1/1}} = 22$ Hz.

Q.9 [375 Hz]

$f = \frac{1}{2\ell}\sqrt{\frac{T}{\rho A}}$ .... (1)

$\frac{m}{A\ell} = \rho$ .... (2)

$\sigma = \frac{T}{A}$ .... (3)

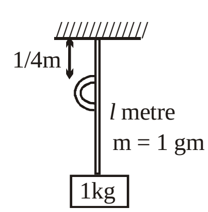
$\therefore f = \frac{T}{2m}\sqrt{\frac{\rho}{\sigma}} = 375$ Hz

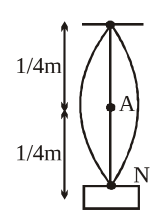
Q.10 [1]

$\mu = \frac{10^{-3}\text{kg}}{l\,(\text{metre})} = \frac{10^{-3}}{l}$ kg / m

$T \approx mg = (1)(10) = 10N$

$\therefore v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{10^{-3}/l}} = \sqrt{10^4/l} = 10^2\sqrt{l}$





$$\frac{1}{4} = \frac{\lambda}{4}$$

$\therefore \lambda = 1m$

$v = \nu\lambda \Rightarrow \qquad 10^2\sqrt{l} = 100\lambda$

$\therefore \ell = 1m$

KVPY
PREVIOUS YEAR'S

Q.1 (D)

Q.2 (B)

$y = A \sin(kx - \omega t)$

at $x = 0.025,\ y = 0.02$

$v = \nu\ \lambda$

$$\lambda = \frac{100}{500} = \frac{1}{5}\ m$$

$$y = 0.02\ = A \sin\left(\frac{2\pi}{\lambda}.\frac{1}{4}\right)$$

$$y = 0.002 = A \sin\left(\frac{5\pi}{2}\right)$$

$\Rightarrow A = 0.02\ m$

$\therefore\ y = 0.02 \sin(kx - \omega t)$

$= 0.02 \sin(10\,\pi \times 0.2 - 1000\pi \times 5 \times 10^{-4}]$

$= 0.02 \sin[2\pi - 0.5\,\pi]$

$$= 0.02 \sin\left(\frac{3\pi}{2}\right) = -0.02\ m$$

Q.3 (B)

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu \times g}{\mu}} = \sqrt{xg}$$

here x = distance from lower end

Now $v^2 = gx \quad \Rightarrow \quad v^2 = 0^2 + 2ax$

$\Rightarrow$ u = 0 and a = g/2 (constant)

so $t = \sqrt{\frac{2x}{a}} \qquad \Rightarrow$ Total time $t \propto \sqrt{L}$

JEE-MAIN
PREVIOUS YEAR'S

Q.1 [2]

$$v = \sqrt{\frac{T}{\mu}}$$

$$\therefore \quad \ell nv = \frac{1}{2}\ \ell nT = \frac{1}{2}\ \ell n\mu$$

$$\%\frac{dv}{v}\ \%\ \frac{1}{2}\ \frac{dT}{T}$$

$$\therefore \quad \%\frac{dv}{v} = \frac{1}{2} \times 4 = 2\%$$

Q.2 [29 N]

$$v = \frac{\omega}{K}\ \ 30\ m/s$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$T = v^2 \times u = \frac{(30)^2 \times (0.325) \times 10^{-3}}{10^{-2}}$$

$= 900 \times 3.25 \times 10^{-2}$

$T = 29.25\ N$

Q.3 (1)

travelling wave function is $f(t \pm x/v)$

Q.4 (4)

$\omega = 2\pi\ f$

$= 1.5 \times 10^3$

$$A = \frac{6}{2} = 3\ cm = 0.03\ m$$

Q.5 [2]

Q.6 (2)

Q.7 [7]

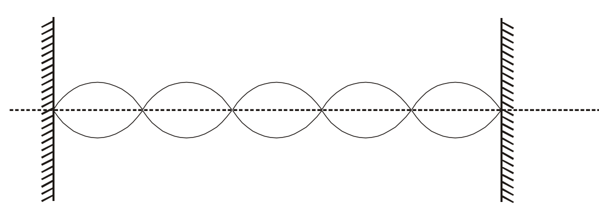
Q.8 [1]

Q.9 [10]

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 (B,C)

(A) There are 5 complete loops.



Total number of nodes = 6

(B) $\omega = 628\ sec^{-1}$

$$k = 62.8\ m^{-1} = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{1}{10}$$

$$v_w = \frac{\omega}{k} = \frac{628}{62.8} = 10\ ms^{-1}$$

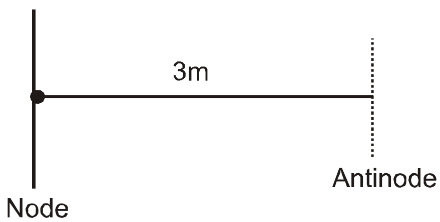
$$L = \frac{5\lambda}{2} = 0.25$$

(C) $2A = 0.01$ = maximum amplitude of antinode

(D) $f = \frac{v}{2\ell} = \frac{10}{2 \times 0.25} = 20$ Hz.

Q.2 (A), (C), (D)

$V = 100$ m/s



Possible modes of vibration

$$\ell = (2n+1)\frac{\lambda}{4}$$

$$\lambda = \frac{12}{(2n+1)}\ m$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{12/(2n+1)} = \frac{(2n+1)\pi}{6}$$

$$\omega = vk = 100\,(2n+1)\frac{\pi}{6} = \frac{(2n+1)50\pi}{3}$$

if $n = 0$ $\quad k = \frac{\pi}{6}$

$$\omega = \frac{50\pi}{3}$$

$n = 1$ $\quad k = \frac{5\pi}{6}$

$$\omega = \frac{250\pi}{3}$$

$n = 7$ $\quad k = \frac{5\pi}{2}$

$\omega = 250\pi$

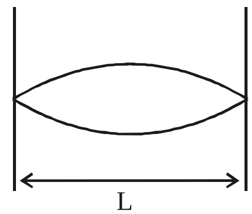
Q.3 (BD)

$V = \sqrt{\frac{T}{\mu}}$, so speed at any position will be same for both pulses, therefore time taken by both pulses will be same.

$\lambda f = v \Rightarrow \lambda = \frac{V}{f} \Rightarrow \lambda \propto V$, since when pulse 1 reaches at A, speed decreases therefore λ decreases. At mid point, magnitude of velocity is same, but direction will be opposite. Hence velocity will be in opposite direction.

Q.4 (A)

For fundamental mode



$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$$

$$f = \frac{V}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

For string (1)

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \Rightarrow (P)$$

For string (2)

$$f = \frac{1}{2L}\sqrt{\frac{T}{2\mu}} = \frac{f_0}{\sqrt{2}} \Rightarrow (R)$$

For string (3)

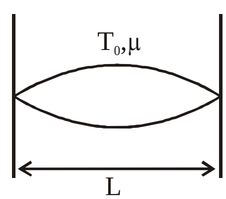
$$f = \frac{1}{2L}\sqrt{\frac{T}{4\mu}} = \frac{f_0}{2} \Rightarrow (Q)$$

Q.5 (A)

For string (1)

Length of string = L_0

It is vibrating in 1st harmonic i.e. fundamental mode.



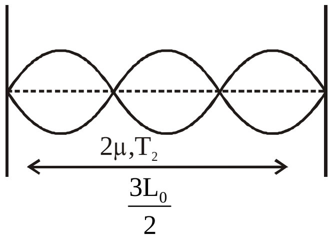
$$f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow (P)$$

For string (2)

Length of string = $\frac{3L_0}{2}$

It is vibrating in IIIrd harmonic but frequency is still f_0.

$$f_0 = \frac{3v}{2L}$$



f_0

$$= \frac{3}{2\left(\frac{3L_0}{2}\right)}\sqrt{\frac{T_2}{2\mu}}$$

$$\Rightarrow f_0 = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

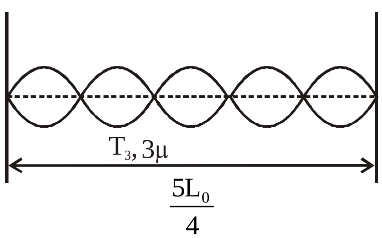
$$\Rightarrow T_2 = \frac{T_0}{2} \Rightarrow (Q)$$

For string (3)

Length of string = $\frac{5L_0}{4}$

It is vibrating in 5th harmonic but frequency is still f_0.

$$f_0 = \frac{5V}{2L}$$



$$\Rightarrow f_0 = \frac{5}{2\left(\frac{5L_0}{4}\right)}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\Rightarrow \frac{2}{L_0}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

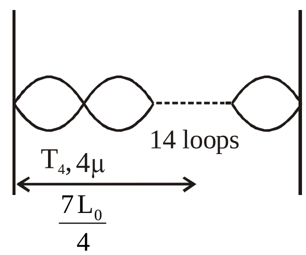
$$T_3 = \frac{3T_0}{16} \Rightarrow (T)$$

For string (4)

Length of string = $\frac{7L_0}{4}$

it is vibrating in 14th harmonic but frequency is still f_0.

$$f_0 = \frac{14v}{2L}$$



$$\Rightarrow f_0 = \frac{14v}{2\left(\frac{7L_0}{4}\right)}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\Rightarrow \frac{4}{L_0}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow (U)$$

ELEMENTARY

Q.1 (1)

(1) Time required for a point to move from maximum displacement to zero displacement is $t = \frac{T}{4} = \frac{1}{4n}$

$$\Rightarrow n = \frac{1}{4t} = \frac{1}{4 \times 0.170} = 1.47 \text{ Hz}$$

Q.2 (1)

(1) The time taken by the stone to reach the lake

$$t_1 = \sqrt{\left(\frac{2h}{g}\right)} = \sqrt{\left(\frac{2 \times 500}{10}\right)} = 10 \sec \left(\text{Using } h = ut + \frac{1}{2}gt^2\right)$$

Now time taken by sound from lake to the man

$$t_2 = \frac{h}{\nu} = \frac{500}{340} \approx 1.5 \text{ sec}$$

$\Rightarrow$ Total time = $t_1 + t_2$ = 10 + 1.5 = 11.5 sec.

Q.3 (4)

$$n = \frac{3600}{60} = 60 \text{ Hz} \Rightarrow \lambda = \frac{\nu}{n} = \frac{960}{60} = 16\,\text{m}$$

Q.4 (3)

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \left(\frac{0.06}{0.03}\right)^2 = \frac{4}{1}$$

Q.5 (1)

$$\nu = \frac{\omega}{k} = \frac{2\pi}{2\pi} = 1\,\text{m/s}$$

Q.6 (2)

$$\nu \propto \sqrt{T} \Rightarrow \frac{\nu_2}{\nu_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow 2 = \sqrt{\frac{T_2}{(273+0)}}$$

$$\Rightarrow T_2 = 273 \times 4 = 1092\,\text{K} = 819\,^\circ\text{C}$$

Q.7 (2)

Speed of sound in gases is given by

$$\nu = \sqrt{\frac{\gamma RT}{M}} \Rightarrow \nu \propto \frac{1}{\sqrt{M}} \Rightarrow \frac{\nu_1}{\nu_2} = \sqrt{\frac{m_1}{m_2}}$$

Q.8 (2)

$$L = 10\log_{10}\left(\frac{I}{I_0}\right) = 30 \Rightarrow \frac{I}{I_0} = 10^3$$

Q.9 (1)

$$\text{Intensity} \propto \frac{1}{(\text{Distance})^2}$$

$$\Rightarrow \frac{I_1}{I_2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

Q.10 (4)

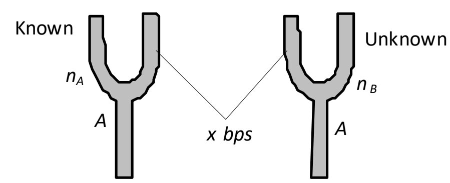
$A_{max} = \sqrt{A_2 + A_2} = A\sqrt{2}$, frequency will remain same i.e. ω

Q.11 (2)

$$a_1 = 5, a_2 = 10 \Rightarrow \frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \left(\frac{5+10}{5-10}\right)^2 = \frac{9}{1}$$

Q.12 (3)

Suppose two tuning forks are named A and B with frequencies n_A = 256 Hz (known), n_B = ? (unknown), and beat frequency x = 4 bps.



Frequency of unknown tuning fork may be

$n_B = 256 + 4 = 260$ Hz

or $= 256 - 4 = 252$ Hz

It is given that on sounding waxed fork A (fork of frequency 256 Hz) and fork B, number of beats (beat frequency) increases. It means that with decrease in frequency of A, the difference in new frequency of A and the frequency of B has increased. This is possible only when the frequency of A while decreasing is moving away from the frequency of B.

This is possible only if n_B = 260 Hz.

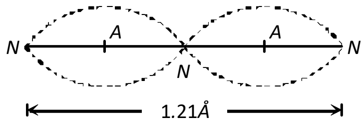
Q.13 (3)

The tuning fork whose frequency is being tested produces 2 beats with oscillator at 514 Hz, therefore, frequency of tuning fork may either be 512 or 516. With oscillator frequency 510 it gives 6 beats/sec, therefore frequency of tuning fork may be either 516 or 504.

Therefore, the actual frequency is 516 Hz which gives 2 beats/sec with 514 Hz and 6 beats/sec with 510 Hz.

Q.14 (1)

$\lambda = 1.21 \text{Å}$



Q.15 (2)

Minimum audible frequency = 20 Hz.

$$\Rightarrow \frac{\nu}{4l} = 20 \Rightarrow l = \frac{336}{4\times 20} = 4.2\,\text{m}$$

Q.16 (4)

Q.17 (3)

$$n' = n\left(\frac{\nu}{\nu - \nu_S}\right) = 1200\times\left(\frac{350}{350-50}\right) = 1400 \text{ cps}$$

Q.18 (3)

Since there is no relative motion between the listener and source, hence actual frequency will be heard by listener.

Q.19 (3)

$$\text{By using } n' = \left(\frac{\nu}{\nu - \nu_s}\right) \Rightarrow 2n = n\left(\frac{\nu}{\nu - \nu_S}\right) \Rightarrow \nu_S = \frac{\nu}{2}$$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (4)

Frequency depends on source not on medium.

Q.2 (1)

$$n = \frac{V}{\lambda} = \frac{.21}{15\times 10^{-3}} = \frac{210}{15}$$

$$V_{max} = A\,\omega = 5\times 10^{-3}\times \frac{210}{15}\times 2\pi$$

$$= 70\times 2\times \frac{22}{7}\times 10^{-3} \qquad = .44\text{m/sec.}$$

Q.3 (1)

$f_1\,\lambda_1 = f_2\,\lambda_2$

$(300)\,(1) = (f_2)\,(1.5)$

$200\text{ Hz} = f_2$

Q.4 (2)

$$\text{time to reach sound wave} = \frac{544}{340}$$

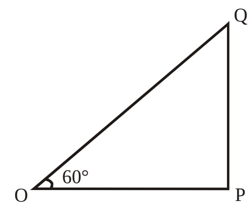
$$\text{time to reach bullet} = \frac{544}{(340-20)} = \frac{544}{320}$$

$$\Delta t = 544\left[\frac{1}{320} - \frac{1}{340}\right] = 544\times\frac{20}{320\times 340} = 0.1 \text{ sec}$$

Q.5 (3)

OQ = 8 × 330 = 2640 m

∴ PQ = OQ sin 60°



$$PQ = 2640\times\frac{\sqrt{3}}{2} = 2286\,\text{m}$$

Q.6 (1)

$$\nu = \sqrt{\frac{\gamma RT}{M}}$$

γ for monoatomic = 1.67

γ for triatomic = 1.3

$$= \sqrt{\frac{\gamma_1}{M_1}\times\frac{M_2}{\gamma_2}},\ = \sqrt{\frac{1.67\times 1.8\times 10^{-2}}{1.3\times 2.02\times 10^{-2}}} = 1.067$$

Q.7 (4)

$$\left(\frac{d}{\nu} - \frac{d}{u}\right) = t,\ d\left(\frac{1}{\nu} - \frac{1}{u}\right) = t \Rightarrow d = \left(\frac{u\nu}{u-\nu}\right)t$$

Q.8 (2)

$$L_1 - L_2 = 10\log\frac{I_1}{I_2}$$

$$3.0103 = 10\log\frac{I_1}{I_2}$$

$$0.30103 = \log\frac{I_1}{I_2} \Rightarrow 2 = \frac{I_1}{I_2}$$

$$\log 2 = 0.30103 \Rightarrow I_1 = 2\times 10^{-4}\,\text{wm}^{-2}$$

Q.9 (1)

$$\text{The speed of sound in air is } v = \sqrt{\frac{\gamma RT}{M}}$$

$\frac{\gamma}{M}$ of H_2 is greatest in the given gases, hence speed of sound in H_2 shall be maximum.

Q.10 (1)

$v_{rms} = \sqrt{\frac{3RT}{M}}$ $v_s = \sqrt{\frac{\gamma RT}{M}}$ $v_{avrage} = \sqrt{\frac{8RT}{\pi M}}$ v_{mp}

$= \sqrt{\frac{2RT}{M}}$

Q.11 (2)

$\beta = 10 \log \frac{I}{I_o}$, $60 = 10 \log \frac{I}{I_o}$

$\beta = 10 \log \frac{8I}{I_o}$

$= 10 \log 8 + 10 \log \frac{I}{I_o}$ $= 30 \log 2 + 60 = 69$ dB.

Q.12 (2)

$I \propto A_0^2$ (Amplitude)

$I \propto \frac{1}{r^2} \Rightarrow A_0 \propto \frac{1}{r} \Rightarrow$ Amplitude $= \frac{A}{r}$

$\Rightarrow \frac{A}{r} \sin(kr - wt)$

Q.13 (4)

$(90 - 40) = 10 \log \frac{I_1}{I_2}$

$5 = \log \frac{I_1}{I_2} \Rightarrow \frac{I_1}{I_2} = 10^5$

Q.14 (4)

$\Delta x = 12 = \frac{\lambda}{2}, \lambda = 24$ cm

$f = \frac{\nu}{\lambda} = \frac{330}{24 \times 10^{-2}} = 1375$ Hz

Q.15 (4)

$\frac{P}{4\pi r^2} = I$ for an isotropic point sound source.

$\Rightarrow$ $P = I.4\pi r^2$

$=$ $(0.008 \text{ w/m}^2)(4.\pi.10^2) = 10.048$

$\cong$ 10 watt.**Ans.**

Q.16 (3)

$B_o = 10 \log \frac{I}{I_o}$ $B_1 = 10 \log \frac{4I}{I_o}$

$= 10 \log 4 + 10 \log \frac{I}{I_o}$

$= 20 \log 2 + B_o = B_o + 6$

Hence when intensity is increased four times, level becomes $(B_0 + 6)$ decibels

Q.17 (4)

$\because$ frequency is same

$\therefore$ energy remains conserved

$\Rightarrow$ Redistribution is stable with time.

Q.18 (2)

path difference $= \pi r - 2r$

$\Delta S = r(\pi - 2)$ $n\lambda = \Delta S$

for constructive interference

$n\lambda = r(\pi - 2)$ $\lambda = \frac{r(\pi - 2)}{n}$ $n = \frac{V}{\lambda} = \frac{Vn}{r(\pi - 2)}$

Q.19 (2)

$\frac{I_1}{I_2} = \frac{4}{1} = 4$

$I_{max} = \left(\sqrt{4} + \sqrt{1}\right)^2 = 9,$ $I_{max} = \left(\sqrt{4} - \sqrt{1}\right)^2 = 1$

$\Delta L = 10 \log \frac{I_{max}}{I_{min}}, = 10 \log \frac{9}{1} = 20 \log 3,$

Q.20 (2)

11^{th} minima from the current minima

$\Delta x = 10\lambda$

$\lambda = \frac{2 \times 5}{10} = 1$ cm

Q.21 (2)

$I_1 = I$ $I_2 = 4I$

$I_A = I_1 + I_2 = 5I$

$I_B = 9I$

$\Delta I = (I_A - I_B) = 4I$

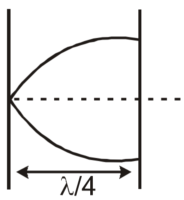
Q.22 (4)

For displacement$\rightarrow$ Phase change π at close end

For pressure $\rightarrow$ No phase change at close end

Q.23 (1)

$$\lambda = \frac{330}{600} = \frac{1}{2}$$

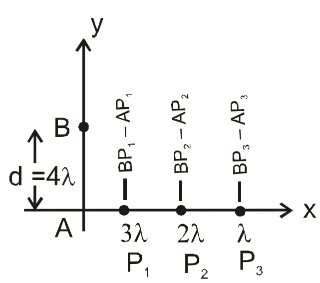


$$\Rightarrow \frac{\lambda}{4} = \frac{1}{8} \Rightarrow 0.125 \text{ m}$$

Q.24 (1)

When a sound wave gets reflected from a rigid boundary, the particles at the boundary are unable to vibrate. Thus, a reflected wave is generated which interferes with the incoming wave to produce zero displacement at the rigid boundary. At these points (zero displacement), the pressure variation is maximum. Thus, a reflected pressure wave has the same phase as the incident wave.

Q.25 (1)



n = 3

Q.26 (3)

$n_1 : n_2 : n_3 = 1 : 2 : 3$

$$\frac{V}{\lambda_1} : \frac{V}{\lambda_2} : \frac{V}{\lambda_3} = 1 : 2 : 3$$

$$\lambda_1 : \lambda_2 : \lambda_3 = 1 : \frac{1}{2} : \frac{1}{3}$$

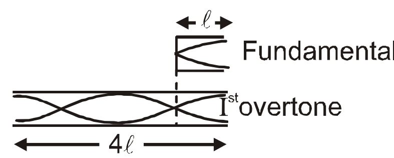
Q.27 (2)

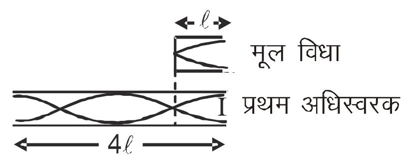
In an open pipe the ends are points of displacement antinodes and hence pressure node. The midpoint (for fundamental mode) is a point of displacement node and hence pressure antinode. (variation of pressure is maximum at pressure antinode and zero at pressure - node).

Q.28 (2)

Closed Open

$$\frac{V}{4\ell_1} = \frac{V}{\ell_2}$$





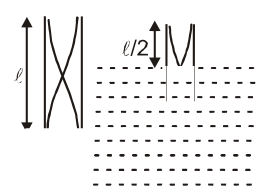
$\ell_2 = 4\ell_1$

$$\ell_1 = \frac{\ell_2}{4} = \frac{50}{4} = 12.5 \text{ cm}$$

Q.29 (3)

Now the tube becomes a closed pipe with length $\ell/2$.

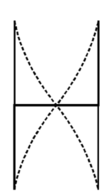
fundamental frequency of B $= \dfrac{v_{sound}}{4(\ell/2)} = \dfrac{v_{sound}}{2\ell}$



which is fundamental frequency of A.

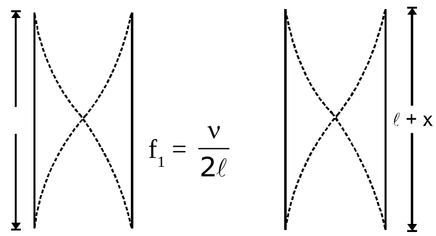
Q.30 (2)

at the middle of the pipe



Q.31 (2)

Q.32 (3)



$f_1 = \frac{\nu}{2\ell}$

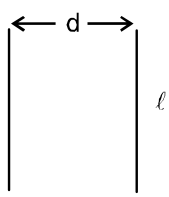
$$f_2 = \frac{\nu}{2(\ell + x)}$$

$$\Rightarrow |\mathbf{f_1} - \mathbf{f_2}| = \frac{\nu}{2\ell} - \frac{\nu}{2(\ell + x)}$$

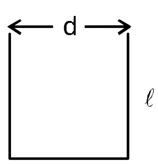
$$= \frac{\nu x}{2\ell(\ell + x)} = \frac{\nu x}{2\ell^2(1 + \frac{x}{\ell})} = \frac{\nu x}{2\ell^2}$$

Q.33 (3)

$$\frac{\lambda_1}{2} = \ell + .6d,\ \upsilon_1 = \frac{V}{\lambda_1}$$



$$\frac{\lambda_2}{4} = \ell + .3d,\ \upsilon_2 = \frac{V}{\lambda_2}$$



$$\frac{\upsilon_2}{\upsilon_1} = \frac{2(\ell + .6d)}{4(\ell + .3d)} = \frac{(\ell + .6d)}{2(\ell + .3d)}$$

Q.34 (2)

Second overtone of open pipe = $\frac{3V}{2\ell_1}$

second overtone of closed pipe = $\frac{5V}{4\ell_2}$

Since, these frequency are same

$$\therefore \frac{3V}{2\ell_1} = \frac{5V}{4\ell_2}$$

$$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{4\times3}{2\times5} = \frac{6}{5}$$

Now, the ratio of fundamental frequencies

$$: \frac{f_1}{f_2} = \frac{\frac{V}{2\ell_1}}{\frac{V}{4\ell_2}} \Rightarrow \frac{2\ell_2}{\ell_1}$$

$= \quad 10 : 6 = 5 : 3$**Ans.**

Q.35 (2)

$$\frac{\lambda}{4} = \ell_1 + e \qquad \text{......(1)}$$

$$\frac{3\lambda}{4} = \ell_2 + e \qquad \text{.......(2)}$$

from (1) and (2) e = 2 cm

Q.36 (1)

Beats

Frequency of tuning fork is 512 Hz. Frequency of sonometer wire either 512 + 6 or 512 – 6

As tension increases frequency of sonometer wire increases $\qquad n \propto \sqrt{T}$

No. of beat reduces. So that frequency of sonometer wire is = 512 – 6 = 506 Hz

Q.37 (2)

|262 – f| = |256 – f| x 2

$\Rightarrow$ (262 – f) = ± (256 – f) x 2

$\Rightarrow$ f = 250, 258 Hz

Unknown Frequency can not be greater than 262 Hz. because no. of beats heard with 262 Hz is more then the no. of beats heard with 256 Hz.

Q.38 (2)

$\sin 2\pi nt + \sin 2\pi(n-1)t + \sin 2\pi(n+1)t$

$$\sin 2\pi nt + 2\sin \frac{[2\pi t(n - 1 + n + 1)]}{2}$$

$$\cos \frac{[2\pi t(n + 1 - n + 1)]}{2}$$

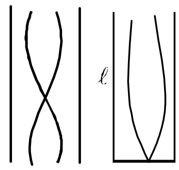
$\sin 2\pi nt + 2\sin 2\pi nt \cos 2\pi t$

$\sin 2\pi nt\ [1 + 2\cos 2\pi t]$

$\Rightarrow f_{beat} = 1$

Q.39 (1)

$$\eta_1 = \frac{V}{2\ell} \quad \eta_2 = \frac{V}{4\ell}$$



no. of beat heard $\qquad n_1 - n_2 = \frac{V}{4\ell} = 4$

if length of pipes are doubled. no of beats heard $n_1' -$

$$n_2' = \frac{V}{8\ell} = \frac{4}{2} = 2$$

Q.40 (2)

$$f = \frac{\nu}{\lambda}$$

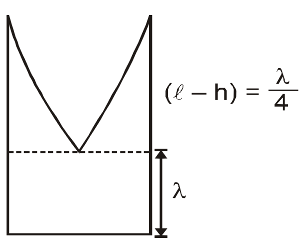
water poured into pipe then
$\lambda \downarrow$ so f $\uparrow$
then Input water = output water
Hence f constant.

Q.41 (3)

$$340 = \frac{340}{4(\ell - h)}$$

$4\ell - 4\lambda = 1$

$$\lambda = \frac{3.8}{4} = 0.95 \text{ m}$$



Q.42 (2)

$$\frac{\ell_1}{\ell_2} = \frac{3\lambda/4}{2\lambda} = \frac{3}{8}$$

Q.43 (4)

$$1750 = \frac{350}{\lambda}$$

$$\lambda = \frac{1}{5} = 20\,\text{cm}$$

should be raised by $\frac{\lambda}{2}$ = 10 cm.

Q.44 (1)

$$fo = \frac{\lambda}{4\ell}$$

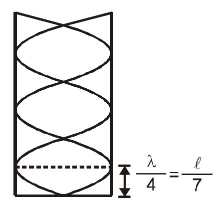
$f = \frac{(2n+1)\nu}{4\ell}$ (2n + 1) overtone

$\Rightarrow$ (2P + 1) overtone

Q.45 (2)

$$\frac{7\lambda}{4} = \ell$$

$$\frac{\lambda}{4} = \frac{\ell}{7}$$



$\therefore$ Amplitude = a

Q.46 (4)
280 ± 10

$$f_s = \frac{\nu}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \Rightarrow T\uparrow, \nu\uparrow, f_s\uparrow \Rightarrow \therefore 290\text{ Hz}$$

Q.47 (2)

$$\frac{\nu}{\lambda_1} - \frac{\nu}{\lambda_2} = 6 \Rightarrow \frac{\nu}{1} - \frac{\nu}{1.02} = 6$$

$$\nu = \frac{6 \times 1.02}{0.02} = 306 \approx 300\text{ m/s.}$$

Q.48 (4)
Doppler effect in Frequency appears when there is relative motion between source and observer

Q.49 (4)
Doppler effect in Frequency depends upon relative velocity between source and observer

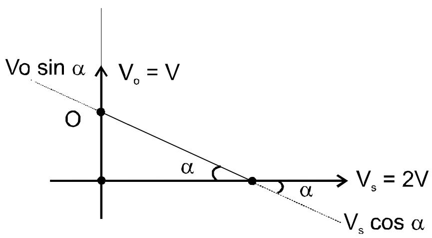
Q.50 (3)

$$n' = \left(\frac{V+v_S}{V}\right)n_r\ ,\ n_r = \left(\frac{V}{V-V_S}\right)n$$

$$n' = \left(\frac{V+V_S}{V-V_S}\right)n = \frac{350+50}{350-50} \times 1.2 = \frac{400}{300} \times 1.2$$

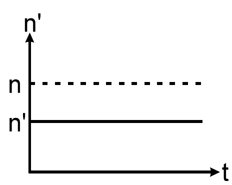
= 1.6 KHz

Q.51 (2)



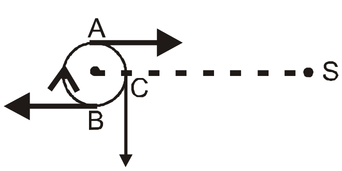
$$n' = \left(\frac{V - V_0 \sin\alpha}{V - V_S \cos\alpha}\right) n \qquad \tan\alpha = \frac{1}{2} \text{ is constant and}$$

n' remains constant and $n' < n$.



so. graph. must be

Q.52 (4)

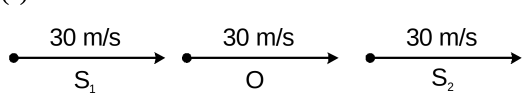


At A observer is moving towards source so $\nu_1 > \nu$.

At B observer is moving away from source so $\nu_2 < \nu$.

At C observer is moving $\perp$ to line joining source and observer so $\nu_3 = \nu$. $\Rightarrow \nu_1 > \nu_3 > \nu_2$

Q.53 (3)



f = 100 Hz

$$f'_{s1} = \left(\frac{\nu - \nu_0}{\nu - \nu_S}\right) 100 = 100$$

$$f'_{s2} = \left(\frac{\nu + \nu_0}{\nu + \nu_S}\right) 100 = 100$$

No Beat.

Q.54 (3)

$$\nu_s - \nu_2 \rightarrow \text{Relative velocity} \quad \Rightarrow t_1 = \frac{a}{\nu_s - \nu_2}$$

Q.55 (3)

$$f' = \left(\frac{\nu + gt}{\nu}\right) f$$

$$f' = f + \frac{gt}{\nu}\ .\ t \Rightarrow \quad f' = 1000 + \frac{g \times 1000}{\nu}\ .\ t$$

t = 30 s.

f' = 2000 Hz $\Rightarrow \nu = 300$ m/s.

Q.56 (1)

$$\lambda = \frac{c+u}{f}$$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (D)

During clapping multiple wave are produced with different wave parameters so wave is resultant of all these wave

Q.2 (B)

Time taken by plate to moves a distance

$$10 \times 10^{-2} = \frac{1}{2} gt^2 = \frac{1}{2}\ 9.8t^2 \Rightarrow t = \frac{1}{7} \text{ sec.}$$

$$f = \frac{\text{no. of vibrations}}{\text{time taken}} = \frac{8}{1/7} = 56 \text{ Hz3}$$

Q.3 (A)

$$160 - 120 = 10 \log \frac{I_1}{I_2}$$

$$10^4 = \frac{I_1}{I_2}\ ,\ I \propto \frac{1}{r^2}$$

$$10^4 = \left(\frac{r_2}{r_1}\right)^2, \quad \frac{r_2}{r_1} = 100$$

$r_2 = 10^4$ m.

Q.4 (B)

$y = A\cos(ax + bt)$

$I_1 = I$

$I_2 = 0.64I$

$I \propto A^2_{net} \Rightarrow 0.64\,A = A^2_{net}$

$A_{net} = \sqrt{I} = 0.8\,A$

Q.5 (B)

First maxima after O will appear when path difference $\Delta S = \lambda$

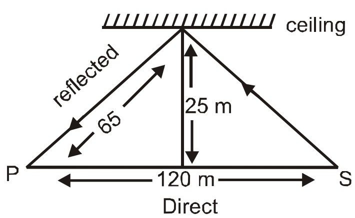
so $AP - BP = \lambda$

$\sqrt{2.4^2 + 1^2} - 2.4 = \lambda\ \lambda = 0.2$

sound velocity = $n\,\lambda = 1800 \times 0.2 = 360$ m/s

Q.6 (A)

Path difference (ΔS) between direct and reflected wave $= 130 - 120 = 10$ m

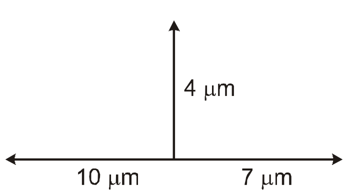


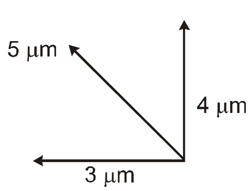
for so constructive interference $\Delta S = n\lambda$

$\lambda = \frac{\Delta S}{n} = \frac{10}{n}$ (n = 1, 2, 3, ...)

$\lambda = 10, \frac{10}{2}, \frac{10}{3}, \frac{10}{4}$ ..... Ans (A)

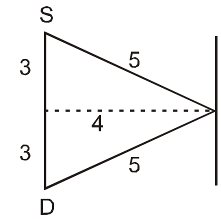
Q.7 (A)





Q.8 (A)

$\Delta x = 10 - 6 = 4 = n\lambda$



$\lambda = \frac{4}{n}$

for $\lambda \to$ max

$n \to$ min = 1

$\lambda = 4$

Q.9 (A)

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \frac{49}{9}$$

$$\Rightarrow \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{7}{3}$$

$$\Rightarrow \sqrt{\frac{I_1}{I_2}} = \frac{10}{4} \Rightarrow \frac{I_1}{I_2} = \frac{25}{4}$$

Q.10 (C)

$\lambda_1 = \lambda_2,\ f_1 = f_2$

$A_1 = A_2\ \Delta\phi = \frac{\pi}{2}$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\frac{\pi}{2}} = \sqrt{A_1^2 + A_2^2}$$

$\nu = f\lambda = f_1\lambda_1 = f_2\lambda_2$

Q.11 (B)

$$f_{fun.} = \begin{cases} \frac{v}{2\ell} & \text{for open pipe} \\ \frac{v}{4\ell} & \text{for closed pipe} \end{cases}$$

$f \propto \sqrt{T}$, but f does not depend on pressure.

for closed pipe $f_{1st\ overtone} = 3f_{fundamental}$.

Q.12 (D)

frequency of two source $n_1 = 50$ $n_2 = 51$

so beat frequency = 1/sec.

Now intensity ratio of maximum & minimum value =

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \left(\frac{16}{8}\right)^2 = \frac{4}{1}$$

Q.13 (D)

To get beat frequency 1, 2, 3, 5, 7, 8, it is possible when other three tuning fork have frequencies 551, 553, 558, etc.

Q.14 (D)

$\ell = 105\text{ cm}$

$$\frac{7\lambda}{4} = 105$$

for Pressure node

$$\frac{\lambda}{4} = \frac{105}{7} = 15\text{ cm}$$

$$\frac{3\lambda}{4} = 45\text{ cm}$$

Q.15 (C)

$$f_1 = \frac{\nu}{4(L+0.6r_1)}$$

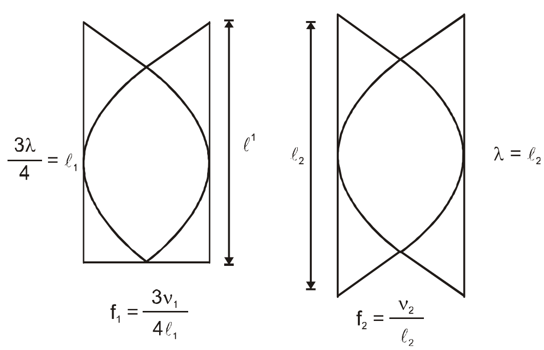
$$f_2 = \frac{\nu}{(L+0.6\times 2r_2)}, \quad f_1 = f_2$$

$$\frac{\nu}{4(L+0.6r_1)} = \frac{\nu}{(L+1.2r_2)}$$

$0.8\, r_1 - 0.4\, r_2 = -L$

$r_2 - 2r_1 = 2.5\,L$

Q.16 (C)



$\because f_1 = f_2$

$$\frac{3\nu_1}{4\ell_1} = \frac{\nu_2}{\ell_2} \Rightarrow \frac{n\nu_1}{4\ell_1} = \frac{m\nu_2}{2\ell_2}$$

$$\Rightarrow \frac{n\nu_1}{4\ell_1} = \frac{m}{2}\left(\frac{3\nu_1}{4\ell_1}\right) \quad \Rightarrow 2n = 3m$$

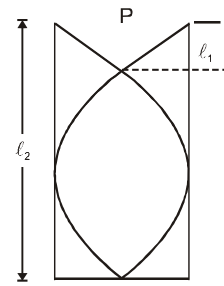
check the options $\quad \Rightarrow n = 9, m = 6.$

Q.17 (C)

At point P so

= zero

$\ell = (\ell_2 - 3\ell_1)$



Q.18 (D)

$$\ell_1 + e = \frac{\lambda}{4} \Rightarrow 40 + e = \frac{\lambda}{4} \quad \text{..........(1)}$$

$$\ell_2 + e = \frac{3\lambda}{4} \Rightarrow 122 + e = \frac{3\lambda}{4} \quad \text{..........(2)}$$

$$\ell_3 + e = \frac{5\lambda}{4}$$

$\therefore (2) - (1)$

$$82 = \frac{\lambda}{2}$$

$$\lambda = 164, e = 1 \Rightarrow \ell_3 = \frac{5}{4}\times 164 - 1$$

$= 205 - 1 = 204$ cm.

Q.19 (C)

For pipe A, second resonant frequency is third harmonic thus $f = \frac{3V}{4L_A}$

For pipe B, second resonant frequency is second harmonic thus $f = \frac{2V}{2L_B}$

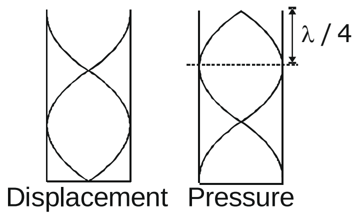
Equating $\frac{3V}{4L_A} = \frac{2V}{2L_B}$

$\Rightarrow L_B = \frac{4}{3} L_A = \frac{4}{3}.(1.5) =$ **2m.**

Q.20 (C)

$$v = \sqrt{\frac{\lambda RT}{M}}, \frac{fc}{fc} = \frac{v_c}{v_D}\times\frac{2\ell/3}{4\ell/3}, \sqrt{\frac{11}{28}}$$

Q.21 (B)



$\frac{3\lambda}{4} = 1.2,\ \frac{\lambda}{4} = \frac{1.2}{3} = 0.4$ from open end.

Q.22 (C)

Open organ pipe $f = \frac{n\nu}{2\ell}$

$\frac{f}{\nu} = \frac{n}{2\ell} = 1.25$ (1)

$\frac{n+1}{2\ell} = 1.75$ (2)

$\frac{n+2}{2\ell} = 2.25$ (3)

from (1) and (2) $\frac{2.5\ell + 1}{2\ell} = 1.75$

$2.5\ \ell + 1 = 3.5\ \ell\ \ell = 1$

Close organ pipe

$f = (2n-1)\ \frac{\nu}{4\ell}$

$\frac{f}{\nu} = \frac{(2n-1)}{4\ell} = 1.25$ (1)

$\frac{2n+1}{4\ell} = 1.75$ (2)

$\frac{2n+3}{4\ell} = 2.25$ (3)

from eq. (1) and (2)

$2n-1 = 5\ \ell$

$2n+1 = 7\ \ell$

–   ––

$-2 = -2\ell\ \ell = 1$ m.

Q.23 (C)

$P = P_0 \cos \frac{3\pi x}{2} \sin 300\ \pi t$

at close end Pr $\rightarrow$ Antinode

at open end Pr $\rightarrow$ node

(A) at x = 0

$P = P_0 \cos 0° = P_0$ Antinode

at x = 0.5 m

$P = P_0 \cos 3\pi = — P_0$ Antinode

(c) at x = 0 antinode – close end.

at x = 2m.

$P_0 \cos 3\pi$ = antinode – close end.

(D) x = 0 antinode – close end.

$x = \frac{2}{3}$ Antindde – close end.

Q.24 (C)

Due to Doppler effect apparent frequency of S_1 will continuously decreases. But apparent frequency of S_2 changes to lower value when it crosses o so best represented graph is.

Ans (C)

Q.25 (B)

$f_1 = f_0 \frac{v_{sound}}{v_{sound} + v_{train}} = \frac{1}{1.2} f_0 \Rightarrow 1.2\ v_{sound} = v_{sound} + v \Rightarrow v = \frac{v_{sound}}{5}$

$f_2 = f_0 \frac{v_{sound} - v_{man}}{v_{sound}} = 0.8 \Rightarrow \frac{f_0}{f_2} = 1.25$

Q.26 (A)

Let original frequency is f

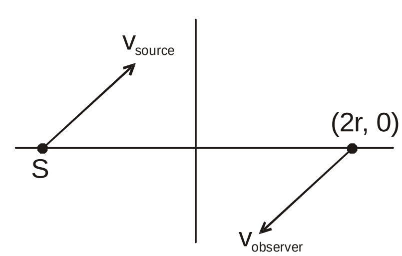
by the concept of Doppler effect

frequency of reflected wave

$f' = \frac{V+u}{V-u} f = \frac{332+12}{332-12} \times f$, $f' - f = 6$,

$\frac{344}{320} f - f = 6 \Rightarrow f = \frac{320 \times 6}{24} = 80$ Hz

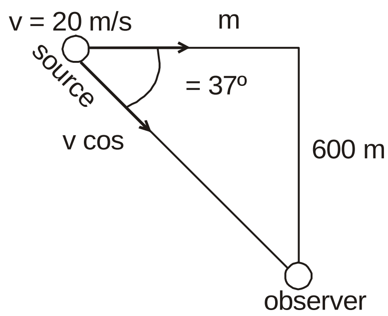
Q.27 (D)



velocity of approach of observer & source decreases, becomes zero and finally becomes velocity of separation.

Hence apparent frequency continuously decreases. ($f_{app} = f$ when $v_{app} = 0$)

Q.28 (C)



$$f = f_0 \frac{v_{sound}}{v_{sound} - v_{source}} \; \frac{V_{/ofu}}{V_{/ofu} - V_{/ofu}}$$

$$= 600 \,.\, \frac{330}{330 - v\cos\theta} = 600 \,.\, \frac{330}{330 - 20\cos 37}$$

$$= 600 \,.\, \left(\frac{330}{314}\right) \simeq 630.5 \text{ Hz.}$$

Q.29 (B)

There is no relative motion between source and observer so frequency remain constant $n = \frac{V}{\lambda_0}$

when wind start blowing in the direction of wave motion then velocity of sound $= V + u_w$

so apparent wave length $\lambda' = \frac{V + u_w}{n} = \frac{V + u_w}{V} \lambda_0$

Ans (B)

Q.30 (A)

frequency of sound for approaching observer $f_a = \frac{c+v}{c} f$

For receding observer $f_r = \frac{c-v}{c} f$

$f_r + f_a = 2f \; f = \frac{f_r + f_a}{2}$

Ans (A)

Q.31 (C)

$$\frac{f_1}{f_2} = \frac{9}{8}$$

$$f_2 = \left(\frac{\nu + \nu_0}{\nu - \nu_s}\right) f_1 \; (\because v_s = v_0)$$

$$\Rightarrow \frac{8}{9} = \frac{\nu + \nu_0}{\nu - \nu_0} \Rightarrow 17\,\nu_0 = -\nu$$

$$\Rightarrow \nu_0 = -20 \text{ m/s} \Rightarrow \quad \nu_0 = 20 \text{ m/s.}$$

Q.32 (C)

Let the velocity of source at mean position is u observer hear maximum frequency when source approaching line from mean positions

$$L_{max} = \frac{c}{c-u}\,\nu$$

and minimum frequency when source reseeding from mean position $L_{max} = \frac{c}{c+u}\,\nu$

velocity at mean positionu = $\sqrt{2gd(1-\cos\theta)}$

$$L_{max} = \frac{c\nu}{c - \sqrt{2gd(1-\cos\theta)}}$$

$$L_{min} = \frac{c\nu}{c + \sqrt{2gd(1-\cos\theta)}}$$

Q.33 (C)

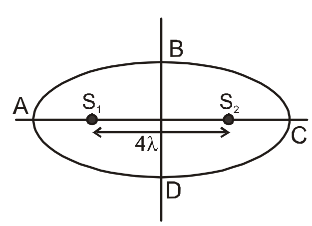
$$n' = n \,.\, \frac{v_{sound}}{v_{sound} - v_{train}} \quad \Rightarrow (n' - n)\, v_{sound} = n' v_{tran}$$

$$n'' = n \,.\, \frac{v_{sound}}{v_{sound} + v_{train}} \quad \Rightarrow (n - n'')\, v_{sound} = n'\, v_{train}$$

$$\Rightarrow \frac{(n'-n)}{n-n''} = \frac{n'}{n''} \Rightarrow n'n'' - nn'' = n'n - n'n''$$

$$\Rightarrow 2n'n'' = (n' + n'')n \quad \Rightarrow n = \frac{2n'n''}{(n'+n'')}$$

Q.34 (A)



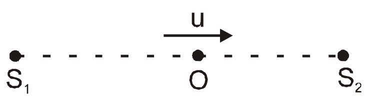
AT B Path difference is O and at A path difference is 4λ. From $n\lambda$ formula there are 3 maxima position between A & B . So total maxima in ellipse = 16

Note $\rightarrow$ if there were circle, rectangle, square instead of ellipse, answer is same.

Q.35 (A)

frequency heard by listener

$$n_1 = \frac{V-u}{V}\frac{V}{\lambda} \quad n_2 = \frac{V+u}{V}\frac{V}{\lambda}$$



beat frequency $= n_2 - n_1 = \frac{2u}{\lambda}$ Ans A

Q.36 (D)

$$\frac{\Delta f}{f} = \left(\frac{\nu}{\nu - \nu_s}\right) - 1 = \frac{1}{10}$$

$$\Rightarrow \frac{\nu}{\nu - \nu_s} = \frac{1}{10} \Rightarrow \qquad \nu_s = \frac{\nu}{11}$$

$$\frac{\Delta f}{f} = \frac{\nu}{\nu + \nu_s} - 1 = \frac{11\nu}{12\nu} - 1 = \frac{1}{12}$$

$$\frac{\Delta f}{f} \times 100 = 8.33\ \%$$

Q.37 (B)

$$f_1 = \left(\frac{\nu}{\nu - \nu_s}\right) n_0$$

$$\frac{f_2}{f_1} = f$$

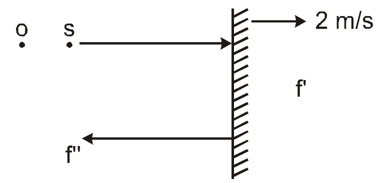
$$f_2 = \left(\frac{\nu}{\nu + \nu_s}\right) n_0 \Rightarrow f = \left(\frac{\nu - \nu_s}{\nu + \nu_s}\right)$$

$$\Rightarrow \nu_s = \left(\frac{1-f}{l+f}\right)\nu \Rightarrow f_1 - f_2 = \frac{\nu n_0[2\nu_s]}{\nu^2 - \nu_s^2}$$

$$\Rightarrow f_1 - f_2 = \frac{1}{2} n_0 \left(\frac{1-f^2}{f}\right)$$

Q.38 (D)

$$f' = \left(\frac{\nu - \nu_0}{\nu}\right) f$$



$$f'' = \left(\frac{\nu}{\nu + \nu_s}\right) f'$$

$$= \left(\frac{\nu - \nu_0}{\nu + \nu_s}\right) f \because \nu_0 = \nu_s$$

$$= \left(\frac{330-2}{330+2}\right) \times 334 = 330\ \text{Hz}$$

Q.39 (A)

$$f' = \left(\frac{\nu + \nu_0}{\nu - \nu_s}\right) f \quad t = 1\ s \Rightarrow \nu_s = 10\ m/s$$

$$f'' = \left(\frac{\nu - \nu_0}{\nu + \nu_s}\right) f \quad \nu_0 = 2 m/s$$

$$f' = \left(\frac{300+2}{300-10}\right)150 \quad = 156$$

$$f'' = \left(\frac{300-2}{300+10}\right) \times 150 = 144$$

$\Rightarrow f' - f'' = 12$ beat

Q.40 (C)

(A) $f = \frac{V}{V + V_B} \times 500$

$$= \frac{350}{350+50} \times 500 = 437.5$$

(B) $f = \frac{V + V_0}{V + V_B} \times 500$

$$= \frac{350+25}{350+50} \times 500, = 468.7 \approx 469\ \text{Hz.}$$

(C) $f_1 = \frac{\nu - \nu_0}{\nu} \times 500 = 464.3\ \text{Hz}$

$(f_B - f_A) = 4.5$ Hz

(D) $f_A = \frac{\nu}{\nu + \nu_B} \times 500 = 437.5\ \text{Hz}$

$$f_B = \frac{\nu - \nu_B}{\nu} \times 500, = 428.6\ \text{Hz}$$

JEE-ADVANCED
MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (C, D)

For stable interference, phase difference should not vary with time.
Hence waves should have same frequency and a constant phase difference.

Q.2 (C, D)

Energy per unit area associated with progressive sound wave $I = 2\pi^2 a^2 n^2 sV$ if we increase amplitude to $\sqrt{2}$ times or frequency to $\sqrt{2}$ times I will be doubled.

So % increment 41% of either amplitude or frequency

Ans – C , D

Q.3 (A, D)

$P_o = B\,k\,S_0, B_1\,k_1\,S_1 = B_2\,k_2\,S_2$ here $B_1 = B_2$ $\because$ of same medium

$$\frac{S_1}{S_2} = \frac{k_2}{k_1} = \frac{\lambda_1}{\lambda_2} = 2\text{Power} = \frac{P_o^2}{2\rho v}$$

P_0 is same for both waves

ρ is same for both waves

v is same for both waves

So, $P_1 = P_2$

Q.4 (A, B, D)

For constructive interference path difference (ΔS)

$\Delta S = n\lambda$ (n = 1, 2, 3, 4.......)

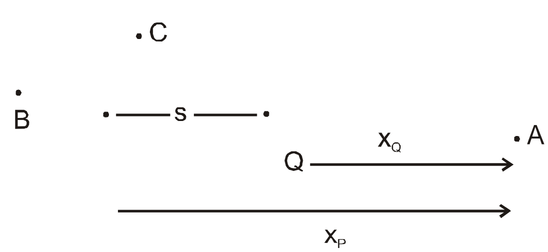
here $\Delta S = 2$ m

So option A & B are satisfied for constructive interference similarly for destructive interference

$$\Delta S = (2n + 1)\,\frac{\lambda}{2}$$

so D option satisfied condition

Q.5 (A, B, D)



Wave emitted from Q $y = A\sin(\omega t - kx_Q)$

Wave emitted from P $y = A\sin\left(\omega t - kx_p + \frac{\pi}{2}\right)$

$$\Delta\phi = \phi_p - \phi_Q = \omega t - kx_p + \frac{\pi}{2} - (\omega t - kx_Q)$$

$$= k(x_Q - x_p) + \frac{\pi}{2}$$

$$= \frac{2\pi}{\lambda}(x_Q - x_p) + \frac{\pi}{2}$$

$$= \frac{\pi}{10}(x_Q - x_p) + \frac{\pi}{2}$$

For A $x_Q - x_P = -5$

$$\Rightarrow \quad \Delta\phi = \frac{\pi}{10}(-5) + \frac{\pi}{2} = 0$$

For B $x_Q - x_P = 5$

$\Rightarrow \quad \Delta\phi = \pi$

For C $x_Q - x_P = 0 \Rightarrow \quad \Delta\phi = \frac{\pi}{2}$

$I_A : I_B : I_C = (I + I + 2I\cos 0) : (I + I + 2I\cos\pi) : \left(I + I + 2I\cos\frac{\pi}{2}\right)$

Q.6 (A, B, C)

When sound wave is reflected from rigid end displacement wave get extra phase at π and pressure wave get no extra phase

So option A, B, C are correct .

Q.7 (B, C)

fundamental frequency of

open pipe, $n_0 = \frac{V}{2\ell_1}$ closed pipe, $n_C = \frac{V}{4\ell_2}$

$$\frac{V}{2\ell_1} - \frac{V}{4\ell_2} = 5 \qquad ...(1)$$

For first overtone $n_0 = \frac{V}{\ell_1}$ $n_C = \frac{3V}{4\ell_2}$

$$\frac{V}{\ell_1} - \frac{3V}{4\ell_2} = 5 \qquad ...(2)$$

on solving (1) and (2) $\ell_1 = \ell_2 \Rightarrow \frac{\ell_1}{\ell_2} = \frac{1}{1}$ Ans (B)

Q.8 (B, D)

making hole at $\frac{\ell}{3}$ length from close end. Pipe start behaving as closed pipe of length $\frac{\ell}{3}$

So new fundamental frequency $n' = \frac{V}{4\ell/3}$

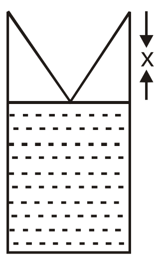
original fundamental frequency $n = \frac{V}{4\ell}$ $n' = 3n$

So option B and D are correct Ans B, D

Q.9 (A, B, C)

$x = \frac{\lambda}{4}$

$\lambda = \frac{V}{n} = \frac{340}{340} = 1$ m



fundamental tone

$x = \frac{1}{4}$ m = 25 cm

For other resonance position

$x = (2n - 1)\frac{\lambda}{4}$

so x = 25, 75, 125 cm....................

Tube length is only 120 cm so get resonance minimum length of water = 120 – 75 = 45

maximum length = 120 – 25 = 95

Distance between two successes nodes

$= \frac{\lambda}{2} = \frac{100}{2} = 50$

Ans A, B, C

Q.10 (B, D)

Fundamental frequency of wave in wire. $n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

here beat frequency $= n_1 - n_2 = \frac{1}{2\ell\sqrt{\mu}}\left(\sqrt{T_1} - \sqrt{T_2}\right)$

beat frequency remain same

T_2 could be increased or T_1 could be decreased

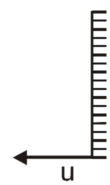
Ans B, D

Q.11 (A, D)

At closed end displacement node and pressure antinode are formed

Q.12 (A, B, C)

By the concept of doppler effect Apparent frequency (no. of waves) striking the wall $= \upsilon = \frac{C+u}{C}\upsilon$



frequency of reflected wave (wall become source) $\upsilon'' = \frac{C+u}{C-u}\upsilon$

Apparent wavelength $= \frac{C}{\upsilon''} = \frac{C(C-u)}{\upsilon(C+u)}$

so option A, B, C are correct Ans = A, B, C

Q.13 (C, D)

High pitch means apparent frequency is increased in reflected wave so there is relative motion between girl and reflection and it is of approaching nature so C and D is correct.

Ans C, D

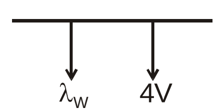
Q.14 (A, C, D)

For observer O_1, $\lambda_1 = \frac{V - V_s}{f} = \frac{V - V/5}{f} = \frac{4V}{5f}$

For O_2, there is change of medium hence at the surface of water, keeping frequency unchanged

$$\frac{V}{\lambda_a} = \frac{4V}{\lambda_w}$$

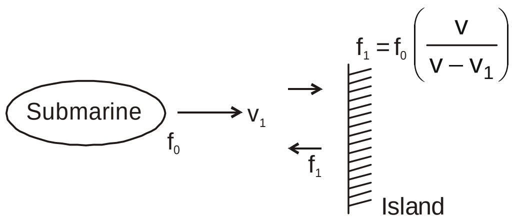
$$\Rightarrow \lambda_w = 4\lambda_a = \frac{16V}{5f}$$



$$f'' = \frac{\text{velo. of wave relative to observer}}{\lambda_w} = \frac{4V + \frac{V}{5}}{\lambda_w}$$

$= \frac{21V}{5} \cdot \frac{5f}{16V} = \frac{21f}{16}$

Q.15 (B)



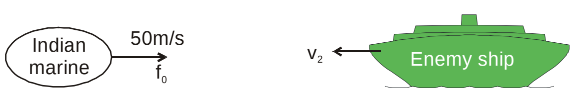
$f'' = f_0\left(\frac{v+v_1}{v-v_1}\right)$, v = 1050

$\left[\frac{f''-f_0}{f_0} = 0.1\right]$

$\frac{f''-f_0}{f_0} = \frac{2V_1}{V-V_1} = 0.1$

$v_1 = 50$ m/sec.

Q.16 (A)



$f'' = f'\left(\frac{v+50}{v-v_2}\right)$ $f' = f_0\left(\frac{v+v_2}{v-50}\right)$

$f'' = f_0\left(\frac{(v+v_2)(v+50)}{(v-v_2)(v-50)}\right) = 1.21f_0$ [21% greater then sent waves]

get $v_2 = 50$ m/sec toward Indian submarine

Q.17 (B)

$\lambda' = \frac{\text{v wrt to observer}}{f'} = \frac{v+v_2}{f_0\frac{(v+v_2)}{(v-50)}} = \frac{v-50}{f_0}$

$\lambda'' = \frac{v+50}{f_0\frac{(v+v_2)(v+50)}{(v-v_2)(v-50)}} = \frac{(v-v_2)(v-50)}{f_0(v+v_2)}$

$\frac{\lambda'}{\lambda''} = \frac{v+v_2}{v-v_2} = \frac{1050+50}{1050-50} = 1.1$

Q.18 (B)

$v = \sqrt{\frac{B}{\rho}} \Rightarrow 1050 = \sqrt{\frac{B}{1000}}$ $B \approx 10^9$ N/m^2

Q.19 (A, B, C)

$D x = pR = nl$

$\lambda = \frac{\pi R}{n}$

$n = 1\ \lambda = \pi R$

$n = 2\lambda = \frac{\pi R}{2}$

$n = 3\lambda = \frac{\pi R}{3}$

$n = 4\lambda = \frac{\pi R}{4}$

Q.20 (A,C,D)

$\Delta x = \frac{3}{4}(2\pi r) - \frac{2\pi r}{4}$

$= \pi r = (2n+1)\frac{\lambda}{2}$

$\lambda = \frac{2\pi r}{2n+1}$

$n = 0\lambda = 2\pi r$

$n = 1\lambda = \frac{2\pi r}{3}$

$n = 2\lambda = \frac{2\pi r}{5}$

Q.21 (B)

$I_{max} = 4I_0\cos^2\frac{\Delta\phi}{2}, \Delta\phi = \frac{\pi}{2}$

$= 4I_0\cos^2\left(\frac{\pi}{4}\right), = 2I_0$

Q.22 (A)

$\Delta x = \pi R = n\lambda$

$\lambda = \frac{\pi R}{n}$ for λ max n = 1

$\lambda = \pi R$

Q.23 (B)

$\Delta x = \pi R = (2n+1)\dfrac{\pi}{2}$

$\lambda = \dfrac{2\pi R}{(2n+1)}$

for $\lambda_{max} n = 0$

$\lambda = 2\pi R \; \lambda = \dfrac{\pi R}{n}$

Q.24 (A)

$\xi = (0.1\text{ mm})\cos\dfrac{2\pi}{80}(y + 1\text{ cm})\cos 2\pi\, 7(400)\, t$

end correction is 1cm. ,so at y = -1 cm.

$\xi = (0.1\text{ mm})\cos\dfrac{2\pi}{80}(-1\text{ cm} + 1\text{ cm}) = (0.1\text{ mm})\cos(0)$

= Antinode

So upper end is open.

at lower end y = 99 cm

$\xi = (0.1\text{ mm})\cos\dfrac{2\pi}{80}(99+1) = 0.01\cos\dfrac{5\pi}{2} = 0$

$\Rightarrow$ Node

tube is closed at lower end

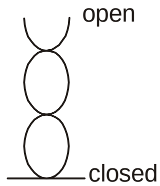
So tube is open closed.

Q.25 (B)

$\dfrac{2\pi}{80} = \dfrac{2\pi}{\lambda}$ So $\lambda = 80$

and effective length of air column = 99 + 1 = 100 cm

So $\dfrac{\ell}{\lambda} = \dfrac{5}{4} \Rightarrow l = 5\dfrac{\lambda}{4}$, so five half loops will be formed



$\ell = 5\left(\dfrac{\lambda}{4}\right)$ so second overtone.

Q.26 (A)

$P_{ex} = -B\dfrac{d\xi}{dx}$

$= (5 \times 10^5) \times (0.1 \times 10^{-3})\dfrac{2\pi}{80}\sin\dfrac{2\pi}{80}(y + 1\text{cm})\cos 2\pi(400)\, t$

$= (125\,\pi\text{ N/m}^2)\sin\dfrac{2\pi}{80}(y + 1\text{cm})\cos 2\pi(400t)$

Q.27 (A)

End correction = (0.3) d = 1 cm

$d = \dfrac{10}{3}$ cm

vol. of tube $= \left(\pi\dfrac{d^2}{4}\right)\ell = \dfrac{\pi}{4}\left(\dfrac{10}{3}\right)^2 \times 100\text{ cm}^3$

(take l = 0.99 m $\approx$ 1 m)

$= \dfrac{10\pi}{36}$ lit

moles $= \dfrac{10\pi}{36 \times 22.4}$ moles (22.4 lt. contains 1 mole

$\dfrac{10\pi}{36}$lt contains $\dfrac{10\pi}{36\times 22.4}$mole)

Q.28 (A) p, q (B) q, s (C) r, (D) s,q

(A) $y = 4\sin(5x - 4t) + 3\cos(4t - 5x + \pi/6)$ is super position of two coherent waves moving in positive direction, so their equivalent will be an another travelling wave.

(B) $y = 10\cos\left(t - \dfrac{x}{330}\right)\sin(100)\left(t - \dfrac{x}{330}\right)$ lets check at any point, say at x = 0,

y = (10 cos t) sin (100 t) at any point amplitude is changing sinusoidally. so this is equation of beats.

(C) $y = 10\sin(2\pi x - 120t) + 10\cos(120t + 2\pi x)$ = superposition of two coherent waves travelling in opposite direction.

$\Rightarrow$ equation of standing waves.

(d) $y = 10\sin(2\pi x - 120\, t) + 8\cos(118t - 59/30\pi x)$ = superposition of two waves whose frequencies are slightly different ($\omega_1 = 120, \omega_2 = 118$)

$\Rightarrow$ equation of Beats.

Q.29 (A) $\to$ (r); (B) $\to$ (s); (C) $\to$ (p); (D) $\to$ (q)

Find all speeds w.r.t. wind and apply doppler effect .

(A) The wavelength of the waves coming towards the observer from source

$= (V - V_W + V_S)/f$

(B) The wavelength of the waves incident on the wall $= (V + V_W - V_S)/f$

(C) The wavelength of the waves coming towards observer from the wall $= (V - V_W - V_D)/f_r$

(D) Frequency of the waves (as detected by O) coming from wall after reflection $= (V - V_W - V_O)f_r/(V - V_W - V_D)$

Q.30 (A) q, r (B) p, t (C) q, s (D) p, t

From standing wave equation

v = (400) (0.8) = 320 m/sec and natural freq. can be

$$\frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell} \ldots\ldots\ldots\ldots$$

80, 240, 400, 560 Hz................

(i) For f = 240, resonance will occur, so only third harmonic will be activated and air column will vibrate violently with purely third harmonic f = 240 Hz.

(ii) For f = 400 Hz, also resonance will occur and air column will vibrate purely with fifth harmonic.

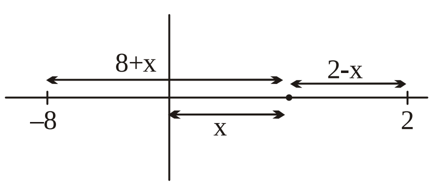
(iii) For f = 320, f = 500 Hz, forcing freq. is not matching with any of the natural freq. So resonance will not occur and air column will vibrate moderately, and there will be some contributions of all harmonics.

NUMERICAL VALUE BASED

Q.1 [4]

$$t = \frac{8+x}{c} = \frac{2-x}{c}$$

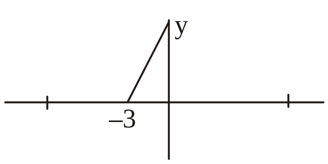
$\Rightarrow -6 = 2x$



x = –3m

$$\Rightarrow t = \frac{5}{c}$$

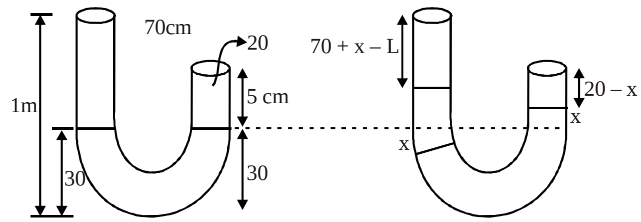
$$5\sqrt{3^2 + y^2}$$



$\Rightarrow$ y = 4

Q.2 [0060]

$$2x \times \rho \times g = L \times \frac{\rho}{2} g$$



$\Rightarrow$ L = 4x

frequency same $\Rightarrow \lambda$ = same

$$20 - x = \frac{\lambda}{4}$$

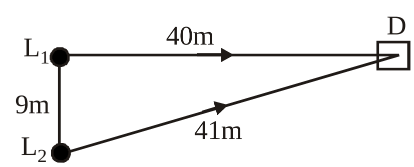
$$70 + 3x = \frac{5\lambda}{4}$$

70 – 3x = 100 – 5x

2x = 30 ; x = 15

L = 4x = 60 cm

Q.3 [0330]



$\Delta x = 1$ m

$n\lambda = 1$

$$f = \frac{C}{\lambda} = C \times n \quad ; n = 1;\ f = 330 \text{ Hz}$$

Q.4 [3I]

Q.5 [0050]

SAN $\Rightarrow$ P Node $\Rightarrow \Delta\phi = \pi$

$$\Delta\phi = \pi \Rightarrow \Delta x = \frac{\lambda}{2}$$

$$\Delta x = \frac{L}{2} + x\left(\frac{L}{2} - x\right)$$

$$2x = \frac{\lambda}{2} \Rightarrow \quad x = \frac{\lambda}{4}$$

$\lambda = 2m \Rightarrow x = 50\,cm$

Q.6 [436]

$f - 440 = 4$ (wrong)

$440 - f = 4$ (correct)

on heating up, tension decreases

$$440 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Q.7 [50]

$f_0 = \frac{v}{4l}$ = Fundametal frequency.

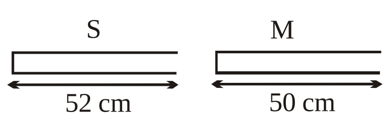
$$f_1 = f_0\left(\frac{v+5}{v}\right); f_2 = f_0\left(\frac{v-5}{v}\right)$$

$$\Rightarrow \frac{165}{160} = \frac{v+5}{v-5} \Rightarrow 33(v-5) = 32\,(v+5)$$

$\Rightarrow 33v - 165 = 32\,v + 160 \Rightarrow v = 325$ m/sec.

$$165 = \frac{v}{4l}\left(\frac{v+5}{v}\right) \Rightarrow l = \frac{330}{4\times165} = 50\,cm$$

Q.8 [6 Hz]



v = 325 m/s

$$\nu_0 = \frac{V}{4L} \qquad \therefore \nu_{10} = \frac{325}{4\times0.52}$$

$$\nu_{20} = \frac{325}{4\times0.50}$$

$$\nu_{10} - \nu_{20} = \frac{325}{4}\left(1 - \frac{1}{0.264}\right)$$

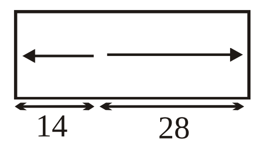
$$= \frac{325}{4}\left(\frac{0.004}{0.260\times0.264}\right)$$

$$= \frac{0.325}{0.260\times0.264} = 6.25\ \text{Hz}$$

Q.9 [16]

$\Delta x = 28\,m = n\lambda$

$$\lambda = \frac{343}{196} = 1.75m$$



$$n = \frac{28}{1.75} = 16$$

$\Rightarrow \Delta\phi = 2\pi \times 16 = 32\,\pi$ **Ans.**

Q.10 [61]

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\Delta\theta)$$

$$= 16 + 25 + 2\,(4)\,(5)\cos\left(\frac{2\pi}{(48/5)}\times1000\right)$$

$= 41 + 20 = 61$

KVPY
PREVIOUS YEAR'S

Q.1 (C)

$f\lambda = v$

$f \propto v$

$$\frac{f_1}{f_2} = \frac{v_1}{v_2}$$

$$\frac{450}{f_2} = \frac{v_1}{1.04v_1}$$

$f_2 = 1.04 \times 450 = 468\,Hz$

Q.2 (B)

S• → u •O

$$f_1 \frac{f_0[v]}{v-u}$$

S• v← •O

$$f_2 = f_0 \frac{[v+u]}{v}$$

$$f_2 - f_1 = f_0\left(\frac{v+u}{v} - \frac{v}{v-u}\right)$$

$$f_2 - f_1 = \frac{-u^2 f_0}{(v)(v-u)} = -ve$$

$\therefore f_1 > f_2$

Q.3 (D)

$$\frac{2\pi}{\lambda}x = \frac{2\pi}{L}x$$

$\therefore \lambda = L = 1.2\ m$

at x = 0, x = L , y = 0

$$\nu = \frac{v}{\lambda} = \frac{300}{1.2} = 250 Hz$$

Q.4 (A)

$$f_1 = f_0\left[\frac{v}{v+v_s}\right]$$

$$f_R = f_1\left[\frac{v-v_s}{v}\right] = f_0\left[\frac{v-v_s}{v+v_s}\right] \text{Fig.} \left[1-\frac{2v_s}{v}\right] f$$

$= 0.990\ f$

Ans. (A)

Q.5 (C)

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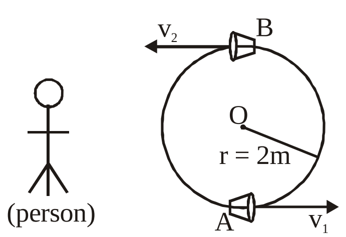


V_1 and V_2 are speed of whistle

$|V_1| = |V_2| = \omega r$

$= 15 \times 2$

$= 30\ m/s$

Maximum frequency heard $\rightarrow$

Here,

F $\rightarrow$ original frequency (540 Hz)

v $\rightarrow$ speed of sound

$v_s \rightarrow$ speed of whistle

$$f_{max} = f\left[\frac{v}{v-v_s}\right]$$

Minimum frequency heard $\rightarrow$

$$f_{min} = f\left[\frac{v}{v-v_s}\right]$$

$$\frac{f_{max}}{f_{min}} = \frac{v+v_s}{v-v_s} = \frac{330+30}{330-30} = \frac{360}{300}$$

$$\frac{f_{max}}{f_{min}} = \frac{6}{5} = 1.2$$

Q.6 (D)

Loudness of sound in decibel dB = 10 $\log_{10}\left(\frac{I}{I_0}\right)$

when intensity of sound become 100 I then new decibel

level = $dB' = 10\log_{10}\left(\frac{100 I}{I_0}\right)$

$dB' - dB = 10 \log_{10} 100$

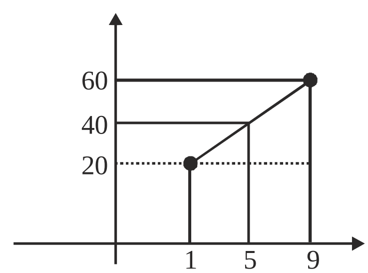
$dB' - dB = 20$

$\therefore$ decibel rise by 20 dB

only one option i.e. 80 to 100 dB match with it.

Q.7 (B)

$$\beta = \log_{10}\left(\frac{I}{I_0}\right)$$



at 5 KHz Hearing capacity = 40 dB

Intensity at 1 KHz

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

$$I = I_0 10^{\left(\frac{\beta}{10}\right)}$$

$(I)_{1\ KHZ} = I_0\ 10^{(20/10)} = I_0(10)^2$

$(I)_{5\ KHz} = I_0\ 10^{(40/10)} = I_0(10)^4$

$$\frac{(I)_{1KHz}}{I_{5KHz}} = \frac{1}{100}$$

Q.8 (C)

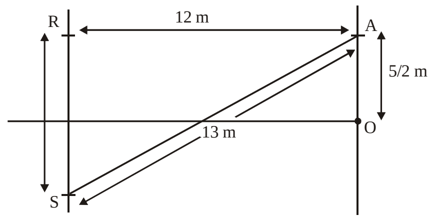
$$\frac{60}{40} = \frac{2d}{v}$$

$$\frac{60}{60} = \frac{2(d-90)}{v} = \frac{2d}{v} - \frac{180}{v}$$

$$1 = \frac{3}{2} - \frac{180}{v} \Rightarrow \frac{180}{v} = \frac{1}{2}$$

$V = 360$ m/s

Q.9 (A)



$$AS - AR = (2n+1)\,\frac{\lambda}{2} = (2n+1)\,\frac{330}{2f}$$

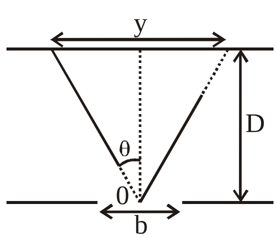
$$13 - 12 = (2n+1)\,\frac{330}{2f}$$

$f = 165\,(2n+1)$

for $n = 1$

$f = 495$ Hz

Q.10 (A)



$$\lambda = \frac{320}{1280} = 0.25\,\text{m}$$

Using concept of differction of wave

$b\sin\theta = 1.22\lambda$

$$\sin\theta = \frac{25}{45} \times 1.22 = 0.678$$

$\tan\theta = 0.93$

$$\tan\theta = \frac{y}{2D}$$

$y \Rightarrow 2D\tan\theta$

$$\text{time to cross this region} = \frac{2D\tan\theta}{\text{speed}}$$

$$\Rightarrow \frac{2\times 8\times 0.93}{1.5} \simeq 9.9\ \text{sec}$$

Q.11 (B)

$r_2 = 80$m, $L_2 = ?$

$r_1 = 8000$ m, $L_1 = 30$ dB

$$L = 10\ \log_{10}\left(\frac{I}{I_0}\right)$$

Intensity due to a point source, $I \propto \frac{1}{r^2}$

$$L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_0}\right) - 10\ \log_{10}\left(\frac{I_1}{I_0}\right)$$

$$L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\ \log_{10}\left(\frac{r_1^2}{r_2^2}\right)$$

$L_2 - 30 = 10\ \log_{10}\ (10^4) = 40$

$L_2 = 70$ dB

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)

$$\lambda = \frac{v}{f} = \frac{336}{504} = 66.66\text{cm}$$

$$\frac{\lambda}{4} = \ell + e = \ell + 0.3d = \ell + 1.8$$

$16.66 = \ell + 1.8$ cm

$\ell = 14.86$ cm

Q.2 (2)

On filing freq. increases so freq. of A would be 335. initially and on filing it would be 338 Hz. So beat freq. become 2 Hz.

Q.3 [16]

frq. observed by A = $f_A = 676$ Hz

$$\text{frq. observed by B} = f_B = \left(\frac{v+u}{v-u}\right)f$$

frq. observed by A after reflection from B is

$$f'_A = \left(\frac{v+u}{v-u}\right) f_B = \left(\frac{v+u}{v-u}\right)^2 f$$

$$\text{Beats frq.}: f_b = f'_A = f_A = \left[\left(\frac{v+u}{v-u}\right) - 1\right]f$$

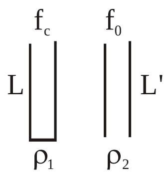
$$= \left[\frac{v^2 + u^2 + 2nv - v^2 - u^2 + 2uv}{(v-n)^2}\right]f$$

$$= \frac{4uv}{(v-u)} \times f \qquad v >>> u$$

$$= \left(\frac{4uv}{v^2}\right)f = \frac{4u}{v} \times f = 4 \times \frac{2}{340} \times 676\text{Hz} = 16\text{Hz}$$

Q.4 [4]

$f_c = f_0$



$$\frac{3V_C}{4L} = \frac{2V_0}{2L'}$$

$$\frac{3V_C}{4L} = \frac{V_0}{L'}$$

$$L' = \frac{4L}{3}\frac{V_0}{V_C} = \frac{4L}{3}\sqrt{\frac{B.\rho_1}{\rho_2 .B}} \text{ (B is bulk modulus)}$$

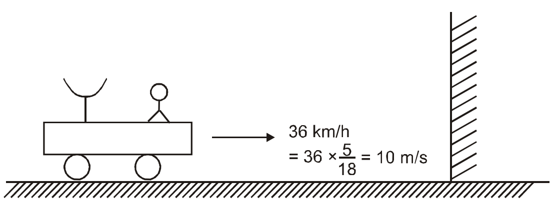
$$= \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

x = 4

Q.5 [132]
Q.6 [34]
Q.7 [1210]
Q.8 [2025]

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (A)



$$f_{inisident} = f_{reflected} = \frac{320}{320-10} \times 8 \text{ kHz}$$

$$f_{observed} = \frac{320+10}{320} f_{reflected}$$

$$= 8 \times \frac{330}{310} = 8.51 \text{ kHz} \approx 8.5 \text{ kHz}$$

Q.2 (A) – p, (B) – p,s, (C) – q,s, (D) – q, r

(A) $\frac{\lambda}{4} = L$, $\lambda = 4L$,
Sound waves are longitudinal waves

(B) $\frac{\lambda}{2} = L$, $\lambda = 2L$
Sound waves are longitudinal waves

(C) $\frac{\lambda}{2} = L$, $\lambda = 2L$
String waves are transverse waves

(D) $\lambda = L$
String waves are transverse waves

Q.3 (B,(D)
At open end phase of pressure wave charge by π so compression returns as rarefraction. While at closed end phase of pressure wave does not change so compression return as compression.

Q.4 (B)

$$\frac{V}{4(\ell + e)} = f$$

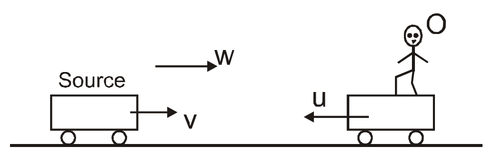
$$\Rightarrow \ell + e = \frac{V}{4f}$$

$$\Rightarrow \ell = \frac{V}{4f} - e$$

here e = (0.6)r = (0.6) (2) = 1.2 cm

$$\text{so } \ell = \frac{336 \times 10^2}{4 \times 512} - 1.2 = 15.2 \text{ cm}$$

Q.5 (A, B)
If wind blows from source to observer



$$f_2 = \left(\frac{(v+w)+u}{(v+w)-u}\right) f_1$$

$\Rightarrow f_2 > f_1$

Q.6 (D)

$$f = \frac{1}{4\ell}\sqrt{\frac{\gamma RT}{M}} \quad \& \quad \frac{\Delta f}{f} = \frac{\Delta \ell}{\ell}$$

(A) $M = 20 \times 10^{-3}$ f = 320 Hz
$\Delta f = \pm 4.5$ Hz Not possible

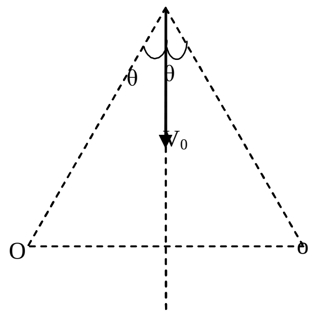
(B) $M = 20 \times 10^{-3}$ f = 253 Hz
$\Delta f = \pm 3.6$ Hz Not possible

(C) $M = 32 \times 10^{-3}$ f = 237 Hz
$\Delta f = \pm 3.4$ Hz Not possible

(D) $M = 36 \times 10^{-3}$ f = 242.8 Hz
$\Delta f = \pm 3.5$ Hz possible

Q.7 (ABC)

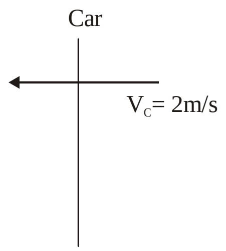
$$\beta = \frac{c + V_0 \cos\theta}{c} f_1 - \frac{c + V_0 \cos\theta}{c} f_2$$



$$= \frac{c + V_0 \cos\theta}{c}\left(f_1 - f_2\right)$$

Hence, (a, b, c)

Q.8 (6)



Frequency observed at car

$$f_1 = f_0\left(\frac{V + V_C}{V}\right)$$

Frequency of reflected sound as observed at the source

$$f_2 = f_1\left(\frac{V}{V - V_C}\right) = f_0\left(\frac{V + V_C}{V - V_C}\right)$$

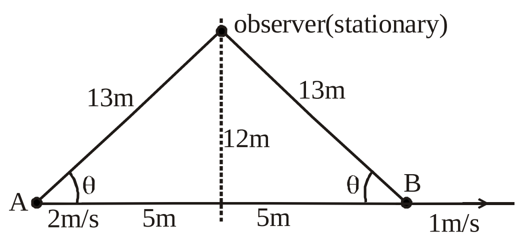
beat frequency = $f_2 - f_0$

$$= f_0\left[\frac{V + V_C}{V - V_C} - 1\right]$$

$$= f_0\left[\frac{2V_C}{V - V_C}\right]$$

$$= 492 \times \frac{2 \times 2}{328} = 6 \text{ beat / s}$$

Q.9 [5.00 Hz]



$$\cos\theta = \frac{5}{13}$$

$$f_A = 1430\left[\frac{330}{330 - 2\cos\theta}\right] = 1430\left[\frac{1}{1 - \frac{2\cos\theta}{330}}\right]$$

$$= 1430\left[1 + \frac{2\cos\theta}{330}\right] \text{ (By bionomial expansion)}$$

$$f_B = 1430\left[\frac{330}{330 - 1\cos\theta}\right] = 1430\left[1 - \frac{\cos\theta}{330}\right]$$

$$\Delta f = f_A - f_B = 1430\left[\frac{3\cos\theta}{330}\right] = 13\cos\theta$$

$$= 13\left(\frac{5}{13}\right) = 5.00 \text{ Hz}$$

Q.10 (A, B, C)

Let n_1 harmonic is corresponding to 50.7 cm & n_2 harmonic is corresponding 83.9 cm.

since both one consecutive harmonics.

$\therefore$ their difference = $\frac{\lambda}{2}$

$$\therefore \frac{\lambda}{2} = (83.9 - 50.7)\text{ cm}$$

$$\frac{\lambda}{2} = 33.2 \text{ cm}$$

$$\lambda = 66.4 \text{ cm}$$

$$\therefore \frac{\lambda}{4} = 16.6 \text{ cm}$$

length corresponding to fundamental mode must be close to $\frac{\lambda}{4}$ & 50.7 cm must be an odd multiple of this length 16.6×3 = 49.8 cm. Therefore 50.7 is 3rd harmonic

If end correction is e, then

$$e + 50.7 = \frac{3\lambda}{4}$$

e = 49.8 – 50.7 = – 0.9 cm

speed of sound, v = fλ

$\therefore$ v = 500 × 66.4 cm/sec = 332.00 m/s

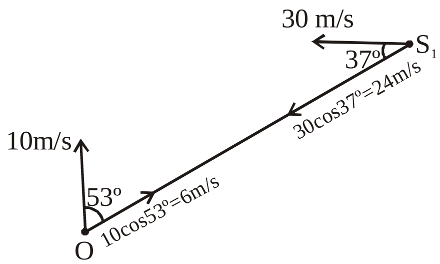
Q.11 [8.12 to 8.13]

Frequency observed by O from S_2

$$F_2 = \frac{330 + 10}{330} \times 120 = \frac{340}{330} \times 120 = 123.63\text{Hz}$$

frequency observed by O from S_1

$$f_1 = \frac{330+6}{330-24}\times 120 = \frac{336}{306}\times 120 \approx 131.76\text{Hz}$$



beat frequency = 131.76 – 123.63 = 8.12 to 8.13 Hz

Q.12 [0.62 to 0.63]

$$f \propto \frac{1}{\ell_1} \Rightarrow f = \frac{k}{\ell_1} \qquad ...(1)$$

($\ell_1 \Rightarrow$ initial length of pipe)

$$\left(\frac{V}{V-V_T}\right)f = \frac{k}{\ell_2}$$ {V_T speed of tuning fork, $\ell_2 \rightarrow$ new length of pipe} ....(2)

(1) ÷ (2)

$$\frac{V-V_T}{V} = \frac{\ell_2}{\ell_1}$$

$$\frac{\ell_2}{\ell_1} - 1 = \frac{V-V_T}{V} - 1$$

$$\frac{\ell_2-\ell_1}{\ell_1} = \frac{V_T}{V}$$

$$\frac{\ell_2-\ell_1}{\ell_1}\times 100 = \frac{-2}{320}\times 100 = -0.625$$

Therefore smallest value of percentage change required in the length of pipe is 0.625

Q.13 (AD)